TECHNICAL UNIVERSITY OF LIBEREC
FAKULTA OF MECHANICAL ENGINEERING
Studentská 2, 461 17 **LIBEREC 1**

MACHINE TOOL DESIGN

1. Static & dynamic stiffness

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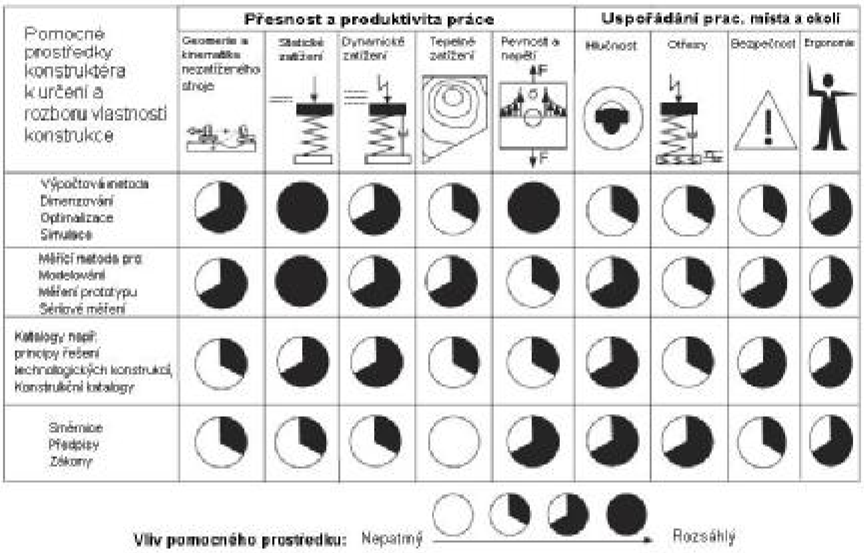
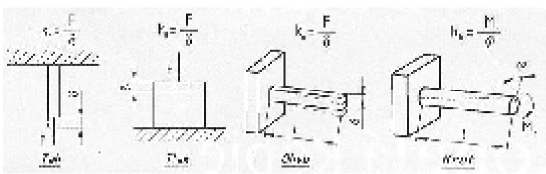
VLIVY V KONSTRUKCI

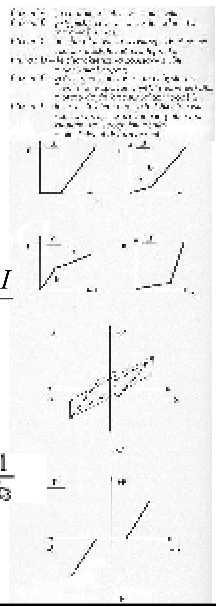


Parametry vlastnosti materiálu

Materiál	Modul pružnosti E [MPa]	Hustota ρ [N.dm^{-3}]	Koef. tepelné roztažnosti α [1/K]	Rozsah pevnosti R_m[MPa]
OCEL	$2,1\,.10^{5}$	7,85	$11,1\,.10^{-6}$	400 ÷ 1300
LITINA	$1,7\,.10^{5}$	7,4	$9,5\,.10^{-6}$	400 ÷ 700
ŠEDÁ LITINA	$0,5 \div 1,1\,.10^{5}$	7,2	$9\,.10^{-6}$	100 ÷ 300
MĚĎ	$1,2\,.10^{5}$	8,95	$16,2\,.10^{-6}$	200 ÷ 400
HLINÍK	$0,7\,.10^{5}$	2,7	$23,8\,.10^{-6}$	120 ÷ 400
MOSAZ	$0,9\,.10^{5}$	8,5	$19\,.10^{-6}$	300 ÷ 700
TITAN	$1,1\,.10^{5}$	4,5	$10,8\,.10^{-6}$	500 ÷ 1200
BETON	$0,2\,.10^{5}$	2,5	$11\,.10^{-6}$	5 ÷ 60

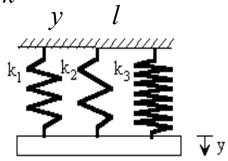
Stiffness Partial and Complex





$$y \frac{F.l}{E.A} \qquad y = \frac{F.l^3}{a.E.l} \qquad j = \frac{M_k.l}{G.I}$$

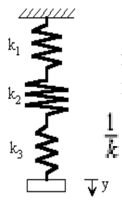
$$k = \frac{F}{y} = \frac{E.A}{l} \qquad k = \frac{F}{y} = \frac{a.E.I}{l^3} \qquad k = \frac{M_k}{j} = \frac{G.I}{l}$$



y = y1= y2= y3

F = F1+ F2+ F3

$$k = k_1 + k_2 + k_3$$

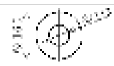
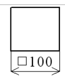
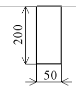
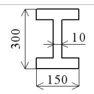


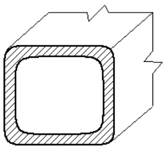
y = y1+ y2+ y3

F = F1= F2= F3

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

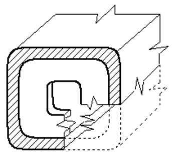
Inertia Moments &Stiffness of different crossectins with equal area (A=100 cm2)

Crossection shape	bending I_O / k	torque I_K / k
	800 / 1	1600 / 1
	2420 / 3	4840 / 3
	4030 / 5	8060 / 5
	834 / 1	1400 / 0,9
	3330 / 4	680 / 0,4
	16000 / 20	143 / 0,09



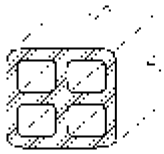
k_O= 100 %

k_K= 1



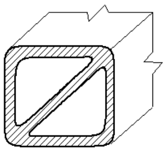
k_O= 1,1

k_K= 1,3 ÷ 1,7



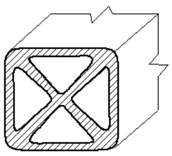
k_O= 1,2

k_K= 2,2



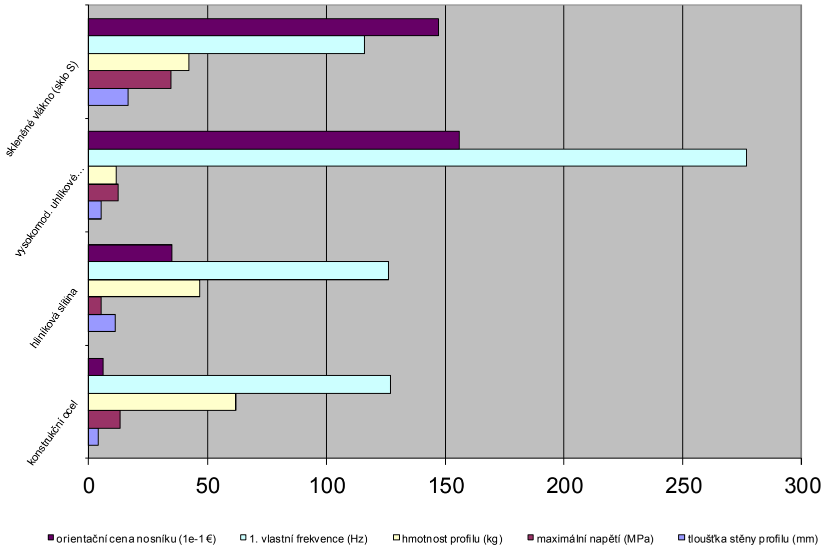
k_O= 1,5

k_K= 3



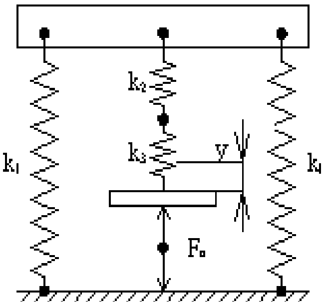
k_O= 1,78

k_K= 3,7



$$k_{ST} = \frac{\text{pressure}}{\text{contact deformatio n in normal direction}}$$

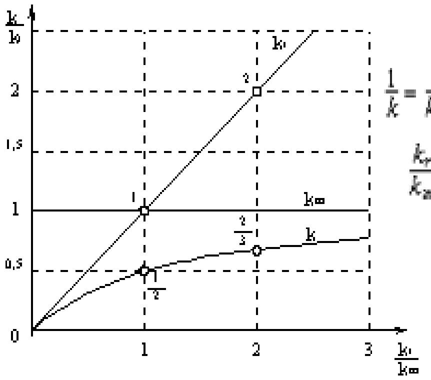
MATERIÁL	**Surface finish**	**Contact Stiffness [MPa.mm-1] Under load**	
		¸ 0,5 [MPa]	**5 [MPa]**
litina / litina	broušeno (Ra= 0,4 ¸ 1 mm)	**0,35**	**0,75**
	lapováno (Ra= 0,08 ¸ 0,16 mm)	**0,5**	**4,2**
ocel / litina	broušeno (Ra= 0,4 mm)	**0,65**	**1,8**
	hoblováno (Ra= 3,2 mm)	**0,4**	**1,25**
ocel / ocel	broušeno (Ra= 0,4 mm)	**0,7**	**2,0**
	hoblováno (Ra= 3,2 mm)	**0,55**	**1,6**



$$k_r = k_1 + k_1 = 2 \cdot k_1$$

$$\frac{1}{k_m} = \frac{1}{k_2} + \frac{1}{k_3} = \frac{k_2 + k_3}{k_2 \cdot k_3}$$

$$\frac{1}{k} = \frac{1}{k_r} + \frac{1}{k_m}$$

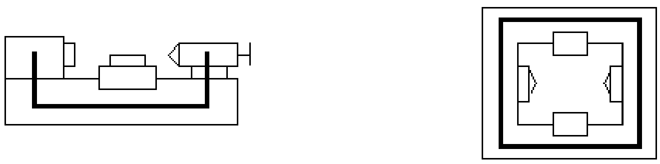


$$\frac{1}{k} = \frac{1}{k_r} + \frac{1}{k_m} = \frac{k_m + k_r}{k_m \cdot k_r} \Rightarrow \frac{k}{k_r} = \frac{k_m}{k_m + k_r} = \frac{1}{1 + \frac{k_r}{k_m}} \approx \frac{\Delta k}{\Delta k_r}$$

$$\frac{k_r}{k_m} = \varphi_1 > 1 \quad k = \frac{k_r}{1 + \frac{k_r}{k_m}} = \frac{\varphi_1}{1 + \varphi_1} \cdot k_m$$

$$k = \frac{j_1 . k_{min}}{(1 + j_1) . j_2} = \frac{1}{j_3} . k_{min}$$

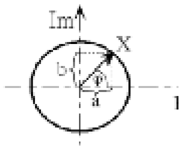
Static Stiffness of Machines



- Enclosed Structure (cabinet) of frame is much more rigid than open frame structure.

Vibration

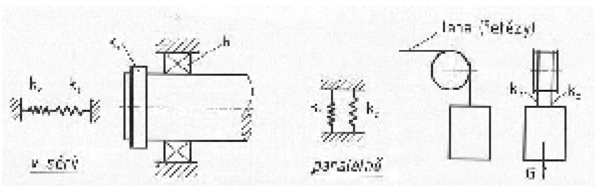
- By path *Circular, Straight*
- By Stiffness Charakteristics *Linar, Nonlinear*
- By Grade of Freedfom *one, more grades*
- By existence of exciting force *free*, *exciting*
- By damping existence *damped with damping constant c*, *undamped*

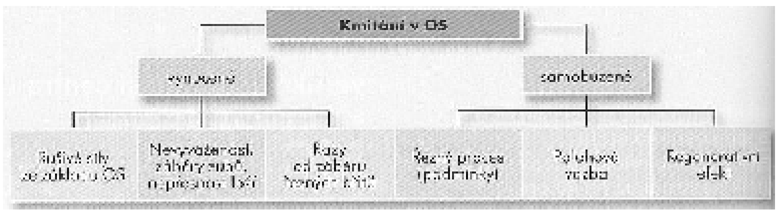


Euler s equation

$$\overline{X} = x.(\cos j + i.\sin j) = x.e^{i.j}$$

VIBRATION ORIGIN, kinds & models





$$k_d = \frac{\text{amplitude of load}}{\text{amplitude of deformatio}}$$

$$k_d^{-1}(R) = \frac{\text{amplitude of deformation}}{\text{amplitude of load}} = RECEPTANCE$$

Origin of vibration are the dynamic forces rised by:

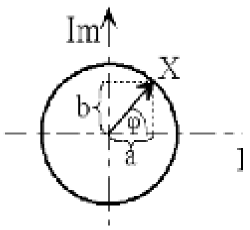
Unbalance of parts with rotary or reversible motion,
inaccuracy of part production,
gaps between parts,
roling parts over itself

Analysis os vibration is made on the basics of complex numbers.

COMPLEX NUMBER $\overline{X} = a + j.b$ **, where absolute value is** $|X| \cong x = \sqrt{a^2 + b^2}$

Real part (Re) Imaginry part (Im)

Complex number is possible to display in Gaus plane as a vector, with x axis – showing real part Re, & y axis – imaginary part Im



$$a = x.\cos j$$

$$b = x.\sin j \qquad j = w.t$$

$\overline{X} = x.(\cos j + i.\sin j) = x.e^{i.j}$ **Eulerův vztah**

$$\overline{X}_t = x.e^{i.t.w} = x.(\cos(w.t) + i.\sin(w.t))$$

Základní dělení kmitavých systémů:

a) By movement - linear (periodical straight movements)
- rotary (circular movements)

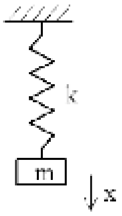
b) By stiffness charakteristics - linear
- nonlinear

c) By degree of freedom – with one degree
- with two degrees (o more degrees)

d) By existence of initiating force – free vibration
- forced vibration

e) By damping existance - damped
- undamped

System of one degree of freedom



- *free vibration undamped*

$$m.\ddot{x} + k.x = 0$$

$$m.s^2.\overline{X}.e^{s.t} + k.\overline{X}.e^{s.t} = 0$$

$$m.s^2 + k = 0 => s_{1,2} = \pm i\sqrt{\frac{k}{m}}$$

$x = \overline{X}.e^{s.t}$

$\dot{x} = s.\overline{X}.e^{s.t}$

$\ddot{x} = s^2.\overline{X}.e^{s.t}$

$\frac{k}{m} = \Omega^2$ W Natural frequency

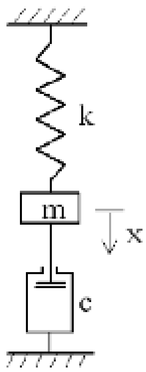
General solution is summation of two particular once:

$$x = x_1 + x_2 = \overline{X}_1.e^{i.\Omega.t} + \overline{X}_2.e^{-i.\Omega.t}$$

Initial conditions are $t=0 \quad x=a \quad \dot{x}=b$

$$x_1 = \frac{1}{2}.(a + i.\frac{b}{\Omega}) \quad x_2 = \frac{1}{2}.(a - i.\frac{b}{\Omega}) \qquad x = a.\cos\Omega.t + \frac{b}{\Omega}.\sin\Omega.t$$

period time: $T = \frac{2.p}{\Omega}$



- *free vibration damped*

$$m.\ddot{x} + c.\dot{x} + k.x = 0$$

$$m.s^2.\overline{X}.e^{s.t} + c.s.\overline{X}.e^{s.t} + k.\overline{X}.e^{s.t} = 0$$

$x = \overline{X}.e^{s.t}$

$\dot{x} = s.\overline{X}.e^{s.t}$

$\ddot{x} = s^2.\overline{X}.e^{s.t}$

$$m.s^2 + c.s + k = 0 \quad => \quad s_{1,2} = \frac{-c \pm \sqrt{c^2 - 4.m.k}}{2.m}$$

$$s_{1,2} = -d \pm \sqrt{d^2 - \Omega^2}$$

$\frac{k}{m} = \Omega^2 \quad \Omega = \sqrt{\frac{k}{m}}$ Natural frequency

$\underline{\delta < \Omega}$

$$s_{1,2} = -d \pm i.\sqrt{\Omega^2 - d^2}$$

$\frac{c}{2.m} = d$ - Damping coefficient; c = 2mδ - damping constant

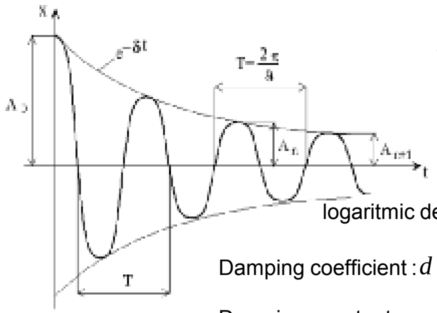
$\underline{\delta > \Omega}$ aperiodical movement (the mass returnes into initial position)

$\underline{\delta = \Omega}$ critical damping c_c. $\quad c_c = 2.m.\Omega$

$\underline{\delta < \Omega}$ vibrations with circular frequency: $\quad J = \sqrt{\Omega^2 - d^2}$

$$x=\overline{X}_1.e^{(-d+i.J)t}+\overline{X}_2.e^{(-d+i.J)t}=e^{-d.t}.(\overline{X}_1.e^{j.J.t}+\overline{X}_2.e^{-i.J.t})$$

Sometimes it is neccessary to find damping coeff. By the outer impuls we start vibration and record the time responce (osciloskop). We may recieve a similar picture as shown



$$A_n = A_1.e^{-d.t} \qquad A_{n+1} = A_1.e^{-d.(t+T)}$$

$$\frac{A_n}{A_{n+1}} = \frac{A_1.e^{-d.t}}{A_1.e^{-d.(t+T)}} = \frac{1}{e^{-d.T}} = e^{d.T}$$

$$\ln\frac{A_n}{A_{n+1}} = \ln e^{d.T} = d.T$$

logaritmic dekrement: $\Delta = d.T$

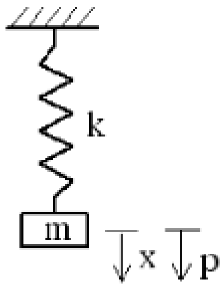
Damping coefficient : $d = \frac{\Delta}{T}$

Damping constant : $c = 2.m.d = \frac{2.m.\Delta}{T}$

Proportional damping <u>D</u> = propotion of system damping constant c over critical damping c_c.

$$D = \frac{c}{c_c} = \frac{2.m.d}{2.m.\Omega} = \frac{d}{\Omega}$$

D = 0,02 0,l



- *<u>Forced vibration undamped</u>*

$$m.\ddot{x}+k.x=p \qquad m.\ddot{x}+k.x=F.e^{j.w.t}$$

Homogenous solution

$$x=\overline{X}_1.e^{j.\Omega.t}+\overline{X}_2.e^{-j.\Omega.t}=a.\cos\Omega.t+\frac{b}{\Omega}.\sin\Omega.t$$

Particular solution

$$m.\ddot{x}+k.x=F.e^{j.w.t} \qquad x=X.e^{j.w.t}$$

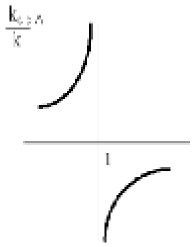
$$\dot{x}=j.w.X.e^{j.w.t}$$

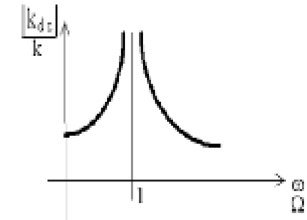
$$-m.w^2.X.e^{j.w.t}+k.X.e^{j.w.t}=F.e^{j.w.t} \qquad \ddot{x}=-w^2.X.e^{j.w.t}$$

Þ $X=\frac{F}{k-m-w^2}=\frac{F}{k-\frac{k}{\Omega^2}.w^2}=\frac{F}{k}.\frac{1}{1-\frac{w^2}{\Omega^2}}=\frac{F}{k}.\frac{\Omega^2}{\Omega^2-w^2}$

Þ $R_{dc}=RECEPTANCE \qquad R_{dc}=\frac{X}{F}=\frac{1}{k}.\frac{\Omega^2}{\Omega^2-w^2}$

Receptance i nrepresenting the dynamic behavior depending on initiating force = systém resonce is ratio of deformation amplitude vers. Amplitude of initiating force.

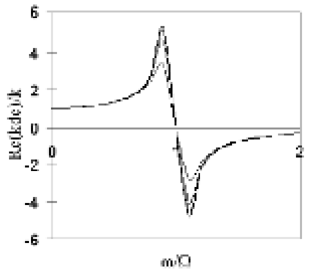


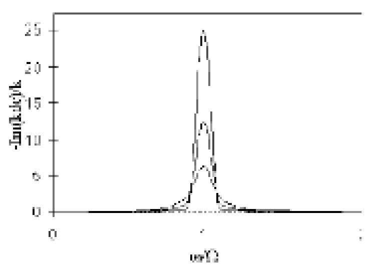


complex rezonance charakteristics *absolute rezonance charakteristics*

Gratest flexibility (deformtion) is when $\omega = \Omega$ or $\frac{w}{\Omega} = 1$ i.e. in the rezonance region, when self frequency is same as frequency of initiating force.

Frequency charakteristics has two parts- real & imaginativ.





- <u>*Forced vibration damped*</u>

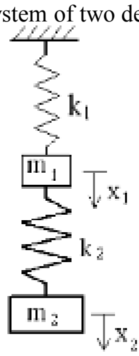
$$x = F\sum_{i=1}^{n}\left(\frac{u_i}{k_{ni}}\cdot\frac{\Omega_i^2}{\Omega_i^2 - w^2 + 2.j.d_i.w}\right)$$

k_{ni} stiffness of i-th part
Ω_i natural frequency of i-th part
u_i Directional coefficient of i-th part
Vibration shape

$$R_{dco} = \frac{u}{k}\cdot\frac{\Omega^2}{\Omega^2 - w^2 + 2.j.D.\Omega.w}$$

Dampers_ principals

System of two degrees of freedom



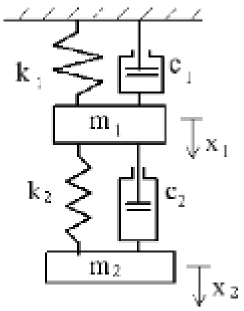
- *free vibration undamped*

$$m_1.\ddot{x}_1 + k_1.x_1 + k_2.(x_1 - x_2) = 0$$
$$m_2.\ddot{x}_2 + k_2.(x_2 - x_1) = 0$$

$$\begin{bmatrix} \left(s^2 + \Omega_{11}^2 + \Omega_{21}^2\right) & -\Omega_{21}^2 \\ \Omega_{22}^2 & \left(s^2 + \Omega_{22}^2\right) \end{bmatrix}$$

where $\Omega_{11}^2 = \frac{k_1}{m_1}$ $\Omega_{21}^2 = \frac{k_2}{m_1}$ $\Omega_{22}^2 = \frac{k_2}{m_2}$

Are partial freequences

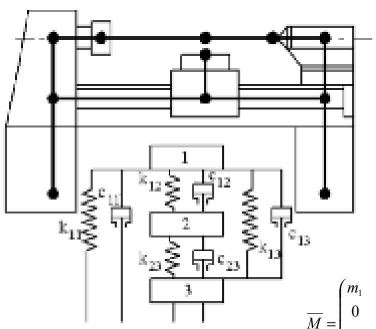


- *free vibration damped*

$$m_1.\ddot{x}_1 + c.\dot{x}_1 + k_1.x_1 + c_2.(\dot{x}_1 - \dot{x}_2) = 0$$
$$m_2.\ddot{x}_2 + c_2.(\dot{x}_2 - \dot{x}_1) + k_2.(x_2 - x_1) = 0$$

$$x_2 = x_{21} + x_{22} = X_{21}.e^{(-d_1 + j.u_1).t} + X_{22}.e^{(-d_2 + j.u_2).t}$$

$J_1 = \Omega_1^2 - d_1^2$ $J_2 = \Omega_2^2 - d_2^2$ Are resulting freequences of parts



$$m_1.\ddot{x}_1 + c_{11}.\dot{x}_1 + c_{12}.(\dot{x}_1 - \dot{x}_2) + ... + k_{11}.x_1 + k_{12}.(x_1 - x_2)... = F_1.e^{j.w.t}$$
$$m_2.\ddot{x}_2 + c_{22}.\dot{x}_2 + c_{21}.(\dot{x}_2 - \dot{x}_1) + ... + k_{22}.x_2 + k_{21}.(x_2 - x_1)... = F_2.e^{j.w.t}$$
$$m_4.\ddot{x}_4 + c_{44}.\dot{x}_4 + c_{41}.(\dot{x}_4 - \dot{x}_1) + ... + k_{44}.x_4 + k_{41}.(x_4 - x_1)... = F_4.e^{j.w.t}$$

$$\overline{M}.\ddot{\overline{x}} + \overline{C}.\dot{\overline{x}} + \overline{K}.\overline{x} = \overline{F}.e^{j.w.t}$$

$$\overline{M} = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix} \quad \overline{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{pmatrix} \quad \overline{K} = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{pmatrix}$$

$\overline{x} = \overline{K^{-1}}.\overline{F}$ $\det(\overline{K} - \Omega^2.\overline{M}) = 0$ For natural freequences $\Omega_1, \Omega_2, ...\Omega_4$

General displacement $\overline{x} = a_1.\overline{X}_1 + a_2.\overline{X}_2 + a_3.\overline{X}_3 + a_4.\overline{X}_4$ Superpozition of individual vibration shapes

$$\overline{x} = \sum_i \left(a_i.\overline{F}_i.\frac{\Omega_i^2}{\Omega_i^2 - w^2 + 2.j.d.w} \right)$$

Summation of vibration amplitudes

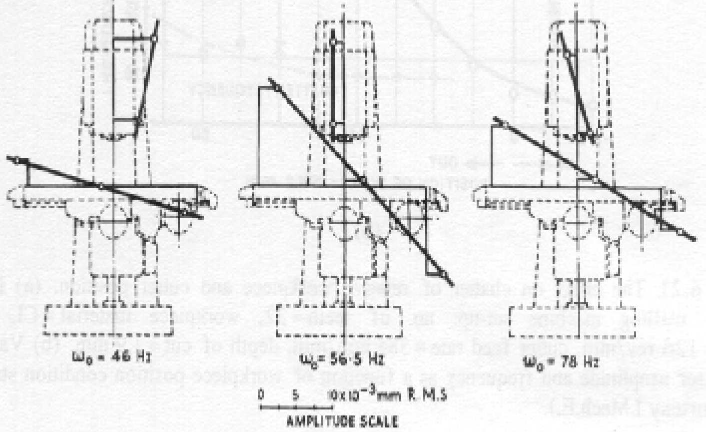


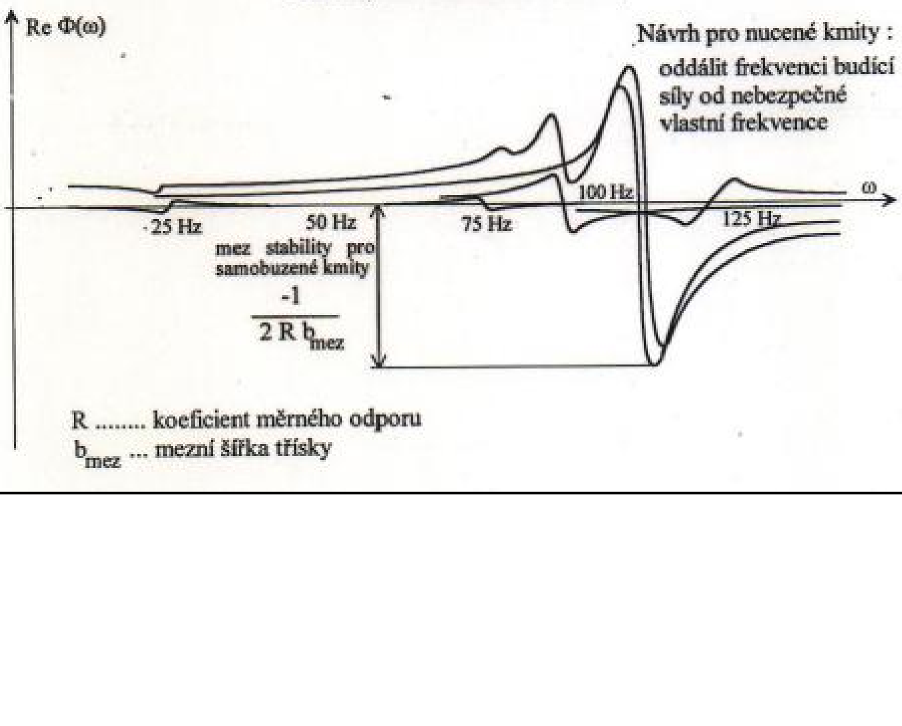
Figure 6.20 Alignment of spindle and table of milling machine in the horizontal modes (courtesy I.Mech.E.).

Pro tlumený systém a vynucené kmity s frekvencí ω přibližně platí :

$$\bar{x} = \sum_i \left(a_i \bar{F}_i \frac{\Omega_i^2}{\Omega_i^2 - \omega^2 + 2 j \delta \omega} \right)$$

- součet tvarů kmitů s vektorem sil přenásobeným dynamickou poddajností.

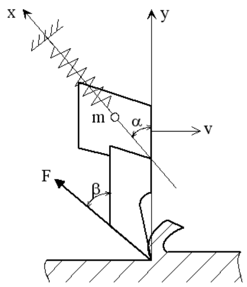
Rozklad frekvenční charakteristiky

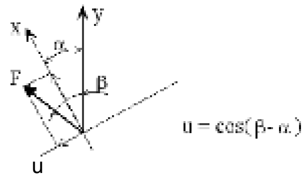


R .... koeficient měrného odporu

b_{mez} ... mezní šířka třísky

$$X = F\sum_{i=1}^{n}\left(\frac{u}{k_{ni}}\cdot\frac{\Omega_i^2}{\Omega_i^2 - w^2 + 2.j.d_i.w}\right) = F\sum_{i=1}^{n} R_m = F.R_{yn}$$

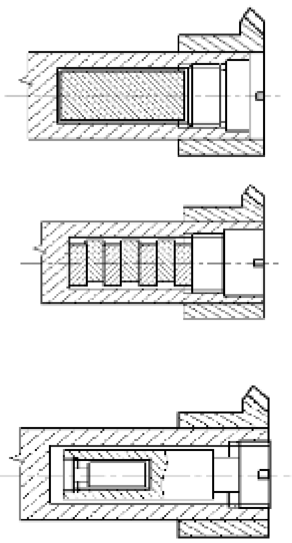




ORIENTED RECEPTANCE

Tlumiče vyvrtávacích tyčí

$$\frac{L}{D} \geq 8 \div 10$$



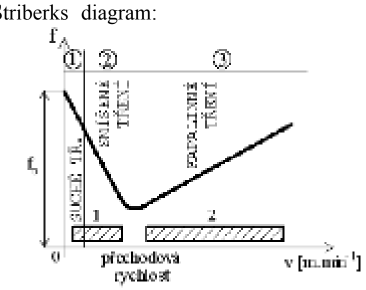
Tlumič –

snažíme se naladit tak, aby tvary kmitů hmoty byly nejlépe v protifázi resp. se zpožděním za budící silou.

SELF VIBRATION (*initiated by the processs itself)*

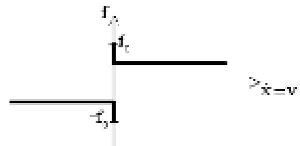
Stick – slip ocurs when heavy loads move very slowly

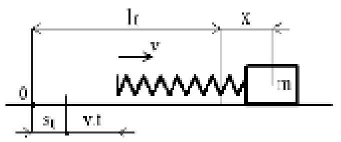
Striberks diagram:



oblast práce 1 - přímočará vedení OS

oblast práce 2 - kluzná rotační uložení OS





l_0 - volná délka pružiny

x - dráha pohybu tělesa

$$m.\ddot{x} + T - k(s_0 + v.t - x) = 0$$

$$m.\ddot{x} + T - k(s_0 + v.t - x) = 0 \qquad m.\ddot{x} + k.x = k.s_0 - T + k.v.t$$

$$\ddot{x} + \frac{k}{m}.x = \frac{\Delta T}{m} + \frac{k}{m}.v.t$$

$$\ddot{x} + \Omega^2.x = \frac{\Delta T}{m} + \Omega^2.v.t$$

Homogenuous solution: $\ddot{x} + \Omega^2.x = 0 \qquad x_h = C_1.\cos\Omega.t + C_2.\sin\Omega.t$

particular solution: $\ddot{x} + \Omega^2.x = \frac{\Delta T}{m} + \Omega^2.v.t$ condition: $x_p = a + b.t$

$\dot{x}_p = b \qquad \ddot{x}_p = 0$

pro $t = 0 \quad x = 0 \quad \dot{x} = 0$

$$0 + \Omega^2.(a + b.t) = \frac{\Delta T}{m} + \Omega^2.v.t \quad \Rightarrow \quad x_p = \frac{\Delta T}{k} + v.t \qquad a = \frac{\Delta T}{m.\Omega^2} = \frac{\Delta T}{k} \quad b=v$$

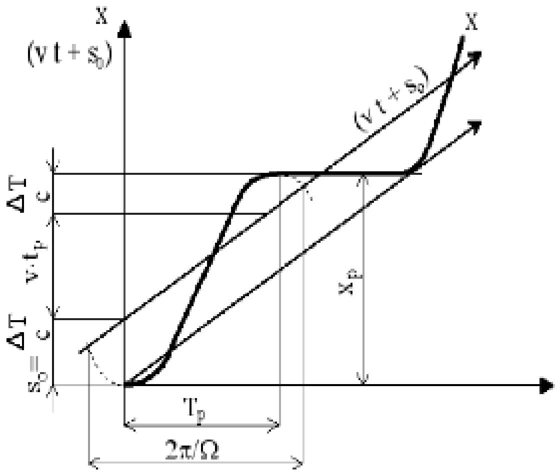
$$X = X_h + X_p = C_1.\cos(\Omega.t) + C_2.\sin(\Omega.t) + \frac{\Delta T}{k} + v.t$$

pro $t = 0 \quad x = 0 \quad \Rightarrow \quad 0 = C_1 + \frac{\Delta T}{k} \qquad C_1 = -\frac{\Delta T}{k}$

$\dot{x} = 0 \quad \Rightarrow \quad 0 = C_2.\Omega + v \qquad C_2 = -\frac{v}{\Omega}$

$$X = -\frac{\Delta T}{k}.\cos(\Omega.t) - \frac{v}{\Omega}.\sin(\Omega.t) + v.t + \frac{\Delta T}{k}$$

Displacement graph in time t



Odstraňování trhavých pohybů: - použitím speciálních maziv, - použitím valivých vedení
- použitím hydrostatického vedení (tlakový olej), - použitím speciálních materiálů na kluzné plochy (teflonu, plastů) o velmi nízkém koeficientu tření