T E C H N I C A L   U N I V E R S I T Y   O F   L I B E R E C
FACULTY OF MECHANICAL ENGINEERING
DEPARTMENT OF MANUFACTURING SYSTEMS AND AUTOMATION

Examples of Machine Parts Calculations

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Content

1 Introduction

The purpose of this educational text is mainly to educate students study of postgraduate Master’s program N2301 – Mechanical Engineering, branch 2302T010 Machine and Equipment Design, field Production Machines, subject Production machines I.

The aim is to provide students with calculation procedure for selected parts and mechanisms of production machines. Additionally, it can be used as guidance for completing individual tasks assigned during exercise classes of Production machines I.

2 Calculation of comparative rate of the machine and calculation of reimbursement time

The goal of this chapter is to demonstrate an example of calculating comparative rate and reimbursement time. These calculations can be used as one of several tools during a decision-making process when modernising machines in a company.

The input is usually following:

- machine price,
- machine establishment costs,
- dimensions of the machine in regard to built-up area,
- machine input,
- amount of working shifts,
- time of manufacturing one part,
- salary and salary cost,
- amortization period, interest rate, maintenance and repair costs, area lease rent, energy price, number of working days in a year.

2.1 Calculation procedure

Formula for calculation of machine comparative rate:

$$S_S = \left(1 + F_O + F_{UR} + F_{UO} + F_P + F_E\right) \cdot \frac{\left(C + N_P\right)}{T_R}$$

F_O machine amortization factor ($F_O = \dfrac{1}{D_O}$), D_O amortisation period machine [years],

F_{UR} interest rate factor ($F_{UR} = \dfrac{U_M}{100}$), U_M interest rate [%],

F_{UO} maintenance and repair factor ($F_{UO} = \dfrac{N_U}{100}$), N_U maintenance and repair costs [%]

F_P area costs factor ($F_P = \dfrac{a \cdot b \cdot C_p}{C + N_P}$)

a, b built-up area dimensions [m], C_P price for lease of 1 m^2 / year,

F_E energy costs factor ($F_E = \frac{P \cdot C_E \cdot T_R}{C + N_P} \cdot k_{pr} \cdot k_v$)

P machine input [kW], C_E price of energy [CZK / kWh],

$k_{pr} = 0.2$ input coefficient, $k_v = 0.5$ machine works coefficient,

C machine price [CZK],

N_P implementation costs (software, tools, installation) [CZK],

T_R effective usage time per year [hours] (for 8 working hours a shift with 80% used time) $T_R = 8 \cdot P_D \cdot 0{,}8 \cdot s$

P_D number of working days in a year,

s amount of shifts,

K_S amount of manufactured pieces a year ($K_S = \frac{T_R \cdot 60}{t_K}$),

t_K time of manufacturing one part [min].

Table 1: Reimbursement time calculation (comparison of two machines)

		Machine I – conventional	Machine II – CNC
time of manufacturing one part	[hours]	t_K^I	t_K^{II}
comparative rate of the machine	[CZK/hour]	S_S^I	S_S^{II}
salary rate	[CZK/hour]	M^I	M^{II}
salary costs	[CZK/hour]	R_M^I	R_M^{II}
service costs	[CZK/hour]	$S_O^I = M^I + R_M^I$	$S_O^{II} = M^{II} + R_M^{II}$
total machine rate	[CZK/hour]	$S_{SC}^I = S_S^I + S_O^I$	$S_{SC}^{II} = S_S^{II} + S_O^{II}$
costs of machining one piece	[CZK/piece]	$N^I = t_K^I \cdot S_{SC}^I$	$N^{II} = t_K^{II} \cdot S_{SC}^{II}$
economic benefit per year	[CZK/year]	$U = \left(N^I - N^{II}\right) \cdot K_S^I$	
reimbursement time	[year]	$T_U = \frac{C^{II} + N_P^{II}}{U}$	

2.2 Assignment examples

The assignment is defined as a real example of conventional machine replacement with a CNC machine. Two students will solve the assignment, while each of them calculates a comparison rate of the machine, and after the students compare their results, they will calculate the reimbursement period. Assignments can be combined by using same shifts.

Table 2: Compared machines

Conventional machine			CNC machine		
P [kW]	a × b [m]	price [CZK]	P [kW]	a × b [m]	price [CZK]
FNGJ 20 milling cutter TOS Žebrák			MH 400P milling cutter MAHO		
2.2	3 x 4	700,000,-	2.2	5 x 4	5,000,000,-
FGS 32/40 milling cutter TOS Žebrák			FNG 63 CNC milling cutter TOS Kuřim		
11	6 x 4	600,000,-	10	6 x 5	1,700,000,-
FN 32 milling cutter TOS Žebrák			UWF 600H milling cutter HERMLE		
3.5	4 x 4	400,000,-	3.3	5 x 5	4,500,000,-
BRH 40A grinding machine Považské stroj.			MFP grinding machine MAGERLE		
5.5	2.5 x 2.5	700,000,-	8.5	3 x 4	5,500,000,-

Other values required for calculation (all of them are the same for all assignments, unless stated otherwise):

- machine establishment costs: 10% of machine price
- time of manufacturing one part: - 144 min for conventional machines,
  - 12 min for CNC machines,

- salary: 50 CZK/hour
- salary costs: 250% of salary
- amortisation period: 10 years,
- interest rate: 2%,
- maintenance and repair costs: 1% of total purchase and implementation cost,
- price of energy: 3.50 CZK/kWh,
- amount of working days in a year: 252 days

3 Draft of boring bar damper

The goal of this chapter is to present calculation of appropriate boring bar damper.

The following values must be determined from the boring bar of the given dimension:

1) Bar deformation (generally),
2) Bar stiffness,
3) Natural bar frequency,
4) Flexibility (receptance).

Additionally, design the bar damper and create a drawing.

The following values are given: ∅D, L/D ratio, n rotation, l_4 damper neck length.

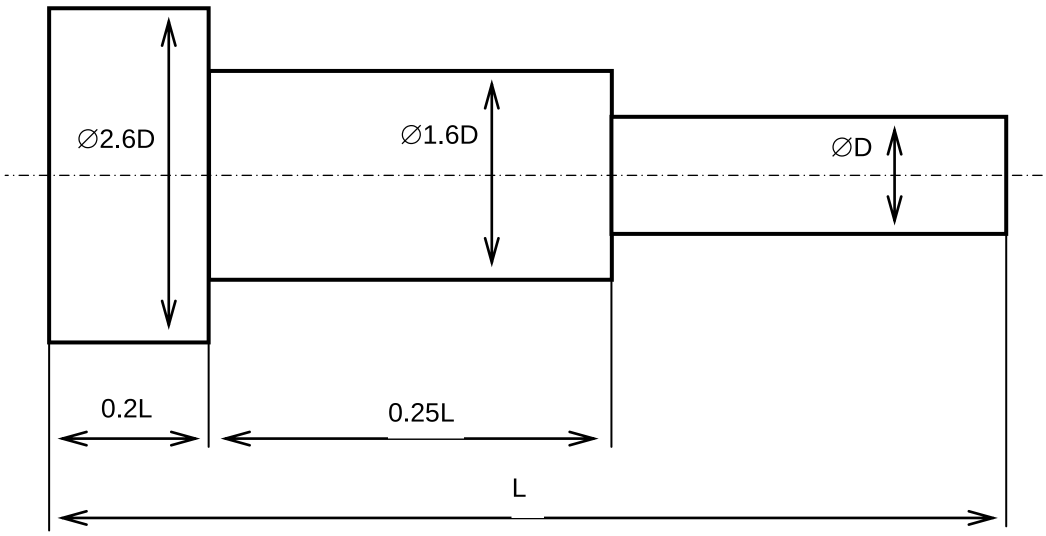


Fig. 1: bar calculation model

3.1 Bar parameter calculation procedure

Formula for calculation of bar deformation:

moment of area ⇒⇒ fictitious beam

$$y \approx M = M_1 + M_2 + M_3$$

$$M_1 = \frac{Fl_1}{EI_1} \cdot l_1 \cdot \frac{1}{2} \cdot \frac{2}{3} l_1$$

....

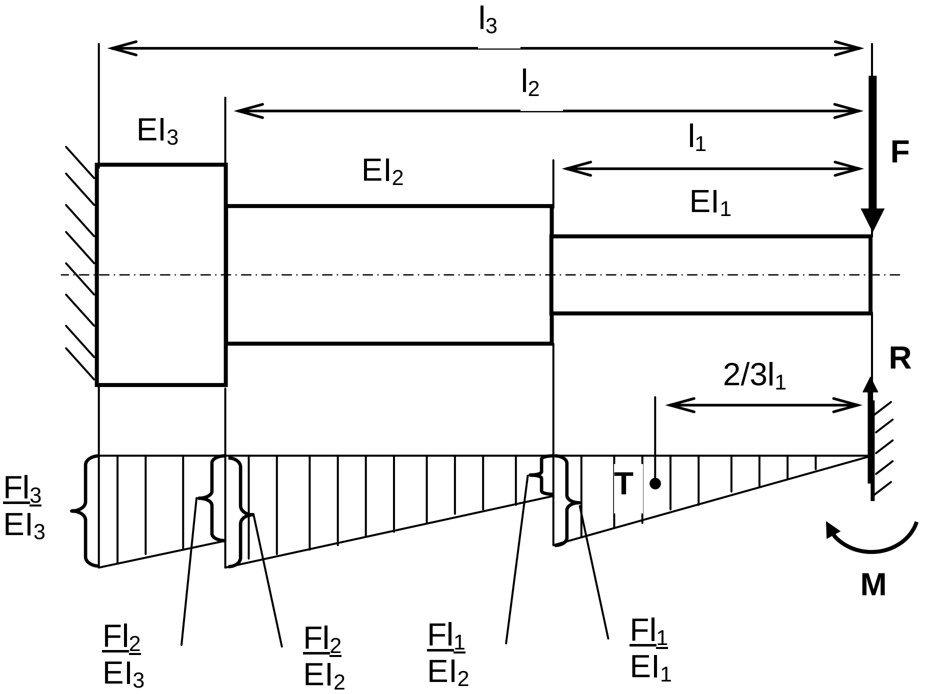


Fig. 2: Replacing the bar with a fictitious beam

$$y = \frac{F \cdot \left(l_3^3 - l_2^3\right)}{3EI_3} + \frac{F \cdot \left(l_2^3 - l_1^3\right)}{3EI_2} + \frac{F \cdot \left(l_1^3\right)}{3EI_1} \quad ; \quad I = \frac{\pi \cdot D^4}{64}$$

Formula for bar stiffness calculation:

$$k = \frac{dF}{dy} = \frac{1}{\left(\frac{l_3^3 - l_2^3}{3EI_3} + \frac{l_2^3 - l_1^3}{3EI_2} + \frac{l_1^3}{3EI_1}\right)}$$

Formula for natural bar frequency calculation:

$$\Omega = \sqrt{\frac{k}{m}} \quad \left[s^{-1}\right] \quad ; \quad f = \frac{\Omega}{2\pi} \quad [Hz]$$

$$m = \rho \cdot V = \rho \cdot \frac{\pi}{4} \cdot \left(D_1^2 \cdot l_1 + D_2^2 \cdot \left(l_2 - l_1\right) + D_3^2 \cdot \left(l_3 - l_2\right)\right)$$

Formula for bar flexibility (receptance) calculation:

$$R_{dc} = \frac{1}{k} \cdot \frac{\Omega^2}{\Omega^2 - \omega^2}$$

3.2 Draft of bar damper

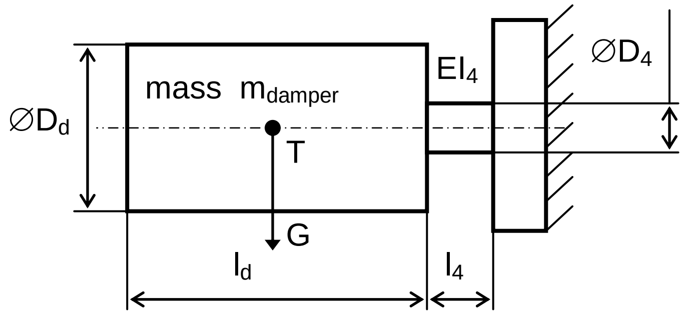


Fig. 3: Damper drawing

Mass and dimensions of the damper must be appropriately selected for dimensions of the boring bar, while preserving the following condition:

$$\underline{\Omega_{bar} = \Omega_{damper}}$$

$$\Omega_d = \sqrt{\frac{k_d}{m_d}} \quad \Rightarrow \quad k_d = m_d \cdot \Omega_d^2$$

$$k_d = \frac{3EI_4}{\left(l_4 + \frac{l_d}{2}\right)^3} \quad \text{(neglecting the mass damper stiffness)}$$

$$\text{Given}: l_4 = 10\ mm \quad ; \quad \text{We choose}: \phi D_d\,;\, l_d\,;\, \rho_d\,;\, E$$

$$m_d \cdot \Omega_d^2 = \frac{3EI_4}{\left(l_4 + \frac{l_d}{2}\right)^3} \quad ; \quad I_4 = \frac{\pi D_4^4}{64} \quad \Rightarrow \quad D_4 = \sqrt[4]{\frac{64}{3\pi} \cdot \frac{m_d \cdot \Omega_d^2}{E} \cdot \left(l_4 + \frac{l_d}{2}\right)^3}$$

$$\text{Range of results}: D_4 = 5 \div 10\ mm$$

3.3 Assignment example

Assignment is different for each student and is based on given L/D ratio of the boring bar (within range L/D > 8), D radius of the boring bar (within range ∅D= 30÷60 mm) and revs (same for all students, e.g. n= 150 rpm). Elastic modulus E= $2.1 \cdot 10^5$ MPa, density ρ= 7 850 kg.m^{-3}, neck length l_4=10 mm.

4 Draft of headstock gearbox

Example of drafting gearbox of headstock for the given values:

- amount of gears - p,
- Φ coefficient,
- min. revs - n_{min} or max. revs - n_{max} [rpm],
- motor revs - n_M [rpm],
- transmitted power - P [kW],

Draw the gearing diagram, revs diagram, logarithmic distribution of torque and kinematic distribution of gearing.

Additionally, calculate the gear modulus and dimensions of gears, then determine the preliminary calculation of shaft dimensions. Determine the real axial distance of shafts.

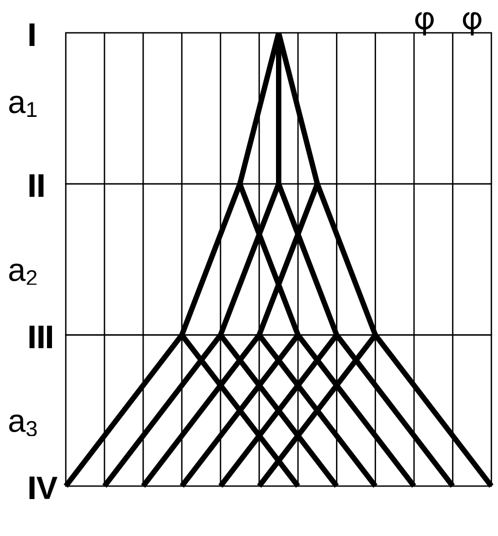
Limit deviations for calculations of gearing: 5%

4.1 Calculation procedure

4.1.1 Gear diagram

E.g. For given amount of gears p = 12

$$p = 12 = k_1 \cdot k_2 \cdot k_3 = 3 \cdot 2 \cdot 2 \quad \Rightarrow \quad \text{tripled gears, coupled gears, coupled gears}$$

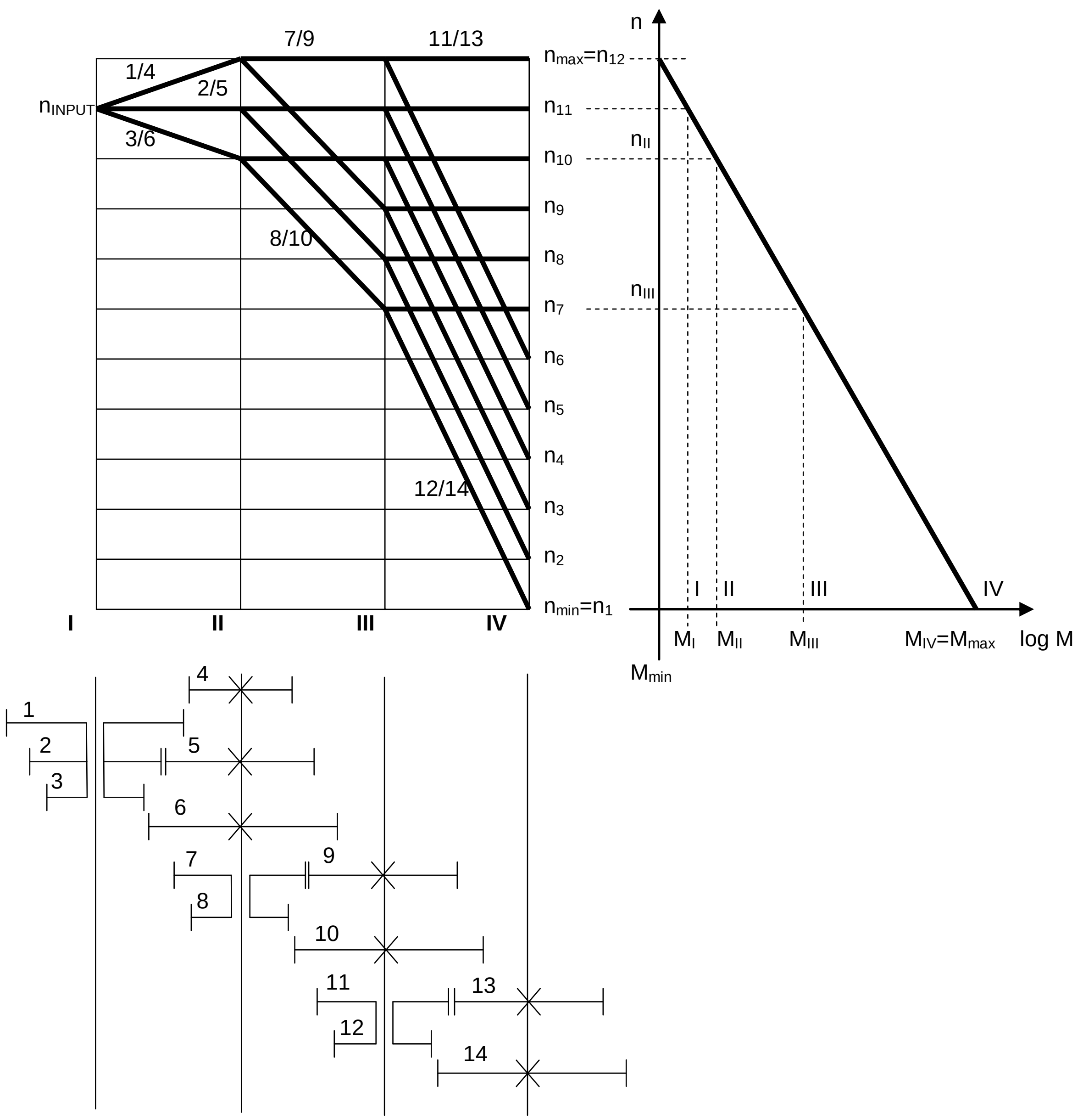


Partial gearing ranges:

$$a_1 = \varphi^{k_1-1} = \varphi^2$$
$$a_2 = \varphi^{k_1\cdot(k_2-1)} = \varphi^3$$
$$a_3 = \varphi^{k_1\cdot k_2\cdot(k_3-1)} = \varphi^6$$

$$k_1 \geq k_2 \geq k_3 \geq k_n$$
$$a_1 \leq a_2 \leq a_3 \leq a_n$$

4.1.2 Speed diagram, torque distribution, kinematic arrangement of gear wheels



Theoretical revs:

$$n_{Ti} = \frac{n_{\max}}{\varphi^{p-i}}, \qquad n_{Ti} = n_{\min} \cdot \varphi^{i-1}, \qquad where\ i \in \langle 1, p \rangle$$

Torque calculation:

$$M_{\min} = \frac{30}{\pi} \cdot \frac{P}{n_{\max}}, \qquad M_{\max} = \frac{30}{\pi} \cdot \frac{P}{\frac{n_{\max}}{\varphi^{p-1}}}, \qquad [Nm, W, \min^{-1}]$$

4.1.3 Calculation of gear wheels dimensions

Number of gear wheels teeth:

The rev diagram implies:

$$\frac{z_1}{z_4} = \varphi \qquad \frac{z_2}{z_5} = 1 \qquad \frac{z_3}{z_6} = \frac{1}{\varphi}$$

$a_{1,4}$, $a_{2,5}$, $a_{3,6}$ axial distances must be identical, i.e. $a_{I,II} = a_{1,4} = a_{2,5} = a_{3,6}$

$$a_{1,4} = \frac{D_1 + D_4}{2} \qquad a_{2,5} = \frac{D_2 + D_5}{2} \qquad a_{3,6} = \frac{D_3 + D_6}{2}, \qquad D_i = m \cdot z_i$$

$$\Rightarrow z_1 + z_4 = z_2 + z_5 = z_3 + z_6, \qquad m = const.$$

Similar with other gearing:

$$\frac{z_7}{z_9} = 1 \qquad \frac{z_8}{z_{10}} = \frac{1}{\varphi^3}, \qquad z_7 + z_9 = z_8 + z_{10}, \qquad m = const.$$

$$\frac{z_{11}}{z_{13}} = 1 \qquad \frac{z_{12}}{z_{14}} = \frac{1}{\varphi^6}, \qquad z_{11} + z_{13} = z_{12} + z_{14}, \qquad m = const.$$

4.1.4 Actual axial distances

$$a_{I,II} = \frac{D_1 + D_4}{2} = \frac{D_2 + D_5}{2} = \frac{D_3 + D_6}{2}$$

$$a_{II.III} = \frac{D_7 + D_9}{2} = \frac{D_8 + D_{10}}{2}$$

$$a_{III,IV} = \frac{D_{11} + D_{13}}{2} = \frac{D_{12} + D_{14}}{2}$$

Calculation of actual revs:

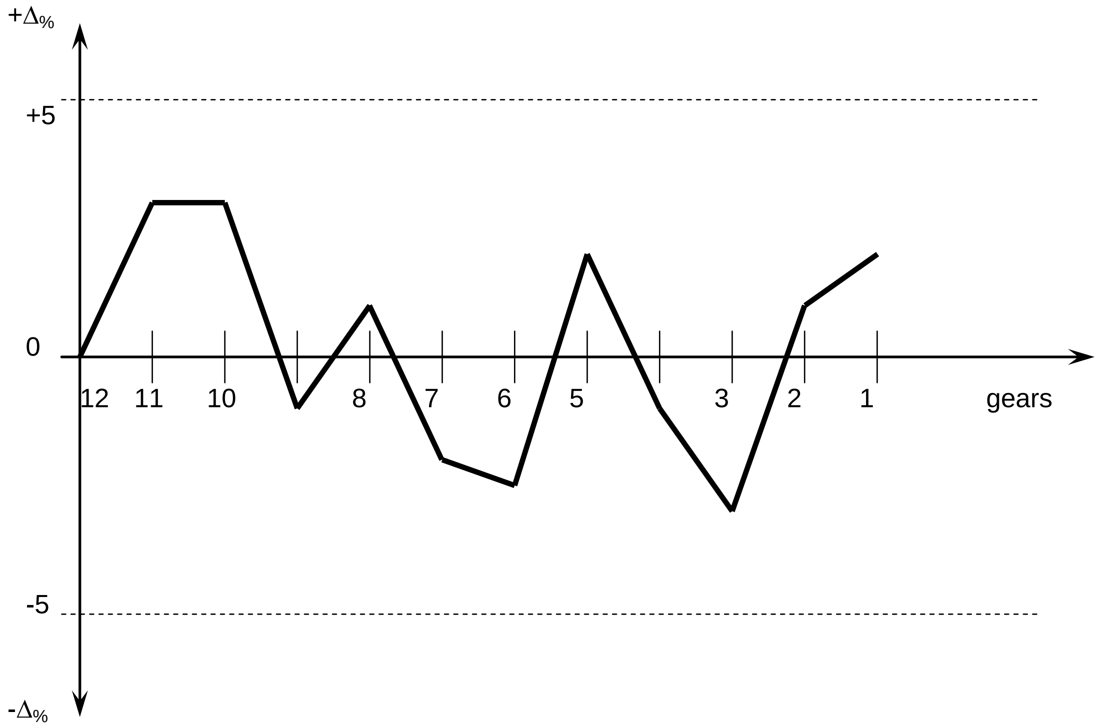
- using the rev diagram and kinematic distribution of gearing:

$$n_{ACT12} = n_{INPUT} \cdot \frac{z_1}{z_4} \cdot \frac{z_7}{z_9} \cdot \frac{z_{11}}{z_{13}}$$

$$n_{ACT1} = n_{INPUT} \cdot \frac{z_{31}}{z_6} \cdot \frac{z_8}{z_{10}} \cdot \frac{z_{12}}{z_{14}}$$

4.1.5 Percentile deviation of revs $\pm\Delta_{\%}$ and its graphical representation

$$\pm \Delta_{\%} = \frac{n_T - n_{ACT}}{n_T} \cdot 100 \quad [\%]$$



4.1.6 Calculation of module

- Preliminary determination of module in relation to load:

$$m = 10 \cdot \sqrt[3]{\frac{2 \cdot M}{c \cdot z \cdot \psi \cdot \pi}} \quad [mm]$$

M … torque [Nm],

c … operating coefficient [MPa],

10 ÷ 20 [MPa] - for first gears,

30 ÷ 45 [MPa] - for final gears,

50 ÷ 60 [MPa] - for high loads,

z ... number of teeth,

ψ … construction parameter $\psi = \frac{b}{m}$; *b* … gear width,

ψ = 4.3 ÷ 13 often ψ = 6 ÷ 8

4.2 Overview of gear wheel parameters of machine-tools

97% gears of machine-tools have gearing maximum ratio of 1 ÷ 4.

number of gear wheels teeth	15	16 ÷ 19	20 ÷ 60	60 and more	
representation in [%]	2	11	83	4	
module	<2	2 ÷ 2.3	2.3 ÷ 5		
representation in [%]	2	8	80		
gear wheels material	11 600 11 700	12 010.9 12 020.9	12 050.6	14 220.9	16 420.6
representation in [%]	up to 1	1.1	2.6	90	4.7

4.3 Approximate calculation of shaft diameters

$$d_I = 10 \cdot \sqrt[4]{\frac{P}{n_{INPUT}}} \quad [cm, kW, \min^{-1}] \qquad d_{II} = 10 \cdot \sqrt[4]{\frac{P}{n_{II}}} \quad [cm, kW, \min^{-1}]$$

$$d_{III} = 10 \cdot \sqrt[4]{\frac{P}{n_{III}}} \quad [cm, kW, \min^{-1}] \qquad d_{IV} = 10 \cdot \sqrt[4]{\frac{P}{n_{\min}}} \quad [cm, kW, \min^{-1}]$$

n_{II} , n_{III} … see rev diagram and logarithmic distribution of torque

5 Optimisation of spindle seating

Calculate the optimal distance of bearings for the given load and characteristic dimension of machine-tool spindles (listed on the attached drawing). Additionally, redraw the spindle drawing in 1:1 ratio.

5.1 Calculation procedure:

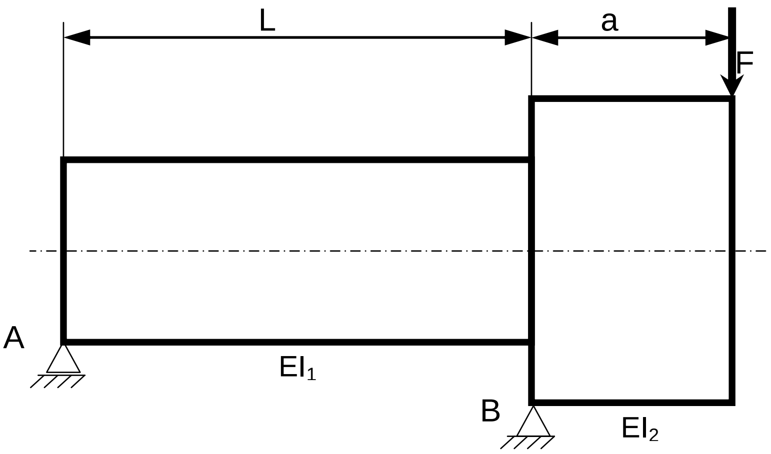
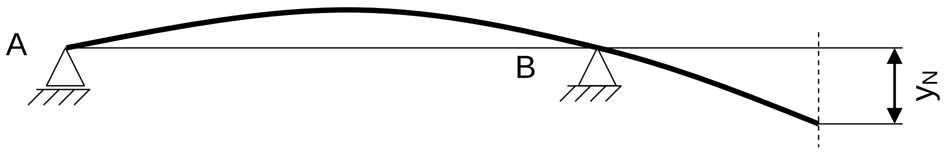


Fig. 4: Calculation model of the spindle

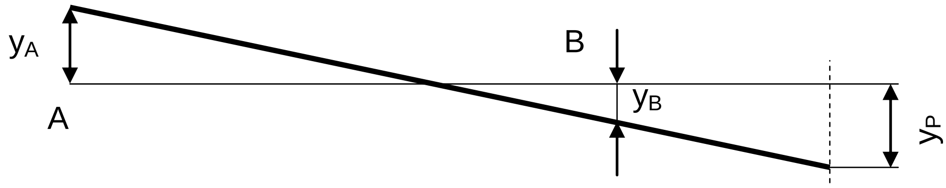
The overall flexure of the spindle is given by sum of spindle and bearing deformation.

1. Flexure of the spindle, on which the F force is applied, if the bearings are stiff:



$$y_N = \frac{Fa^2L}{3EI_1} + \frac{Fa^3}{3EI_2}$$

2. Flexure of the spindle if the bearings are pliable and the spindle is stiff:



$$\frac{y_A + y_P}{a + L} = \frac{y_A + y_B}{L} \qquad \Rightarrow \qquad y_P = \frac{(y_A + y_B)\cdot(a + L)}{L} - y_A$$

y_A , y_B pliable bearing deformation is determined using the following table:

Type of bearing	Loading method	
	$\delta_a = 0$	$\delta_r = 0$
Adjustable ball bearings	$\delta_r = \frac{70 \cdot 10^{-5}}{\cos\alpha} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$	---------------
Radial ball bearings	$\delta_r = 44 \cdot 10^{-5} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$	---------------
Oblique ball bearings	$\delta_r = \frac{44 \cdot 10^{-5}}{\cos\alpha} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$	$\delta_a = \frac{44 \cdot 10^{-5}}{\sin\alpha} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$
Bearings with rectilinear contact on both rings	$\delta_r = \frac{8 \cdot 10^{-5}}{\cos\alpha} \cdot \frac{Q^{0,9}}{L_a^{0,8}}$	$\delta_a = \frac{8 \cdot 10^{-5}}{\sin\alpha} \cdot \frac{Q^{0,9}}{L_a^{0,8}}$
Bearings with rectilinear contact on one ring and point contact on the other	$\delta_r = \frac{22 \cdot 10^{-5}}{\cos\alpha} \cdot \frac{Q^{3/4}}{L_a^{1/2}}$	$\delta_a = \frac{22 \cdot 10^{-5}}{\sin\alpha} \cdot \frac{Q^{3/4}}{L_a^{1/2}}$
Axial ball bearings	---------------	$\delta_a = \frac{52 \cdot 10^{-5}}{\sin\alpha} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$
Rolling element force	$Q = \frac{5 \cdot F_r}{i \cdot z \cdot \cos\alpha}$	$Q = \frac{F_a}{z \cdot \sin\alpha}$

i … amount of rolling element rows,

z … amount of rolling elements in one row,

δ_r … deformation in radial direction [mm],

δ_a … deformation in axial direction [mm],

D_W … radius of the rolling element [mm],

L_a … effective length of the rolling element [mm],

α … angle of contact,

F_r … load in radial direction [N],

F_a … load in axial direction [N],

Q … force applied on rolling element [N].

In case the dimensions of rolling element and their amount are not listed in the catalogue, they can be approximately calculated:

$$D_W = q_1 \cdot (D - d)$$

$$z = q_2 \cdot \frac{D + d}{D_W}$$

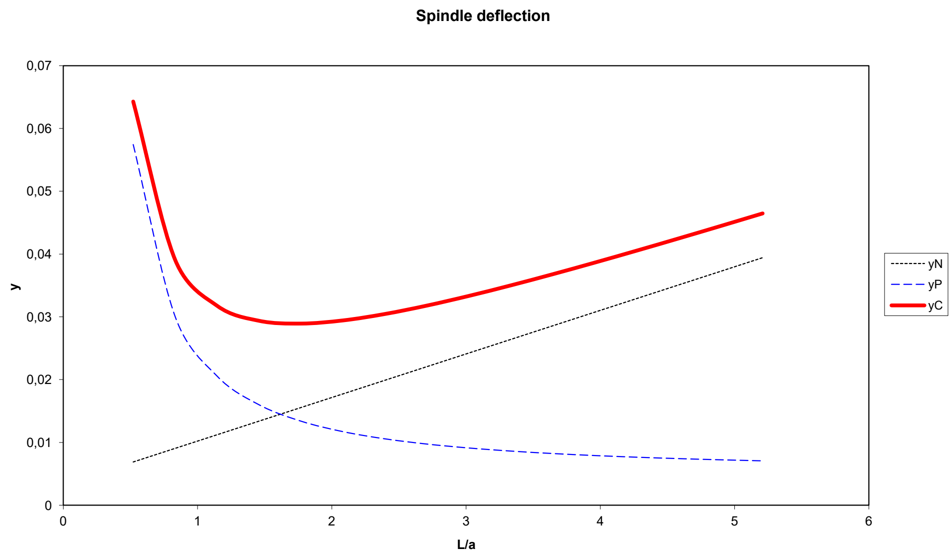
$$L_a = 1{,}4 \cdot D_W$$

Type of bearing	q_1		q_2	
	from	to	from	to
Radial bearings				
Ball, one-row	0.216	0.330	0.890	0.990
Ball, two-row	0.200	0.280	1.190	1.390
Oblique ball bearings, one-row	0.250	0.320	1.240	1.400
Oblique ball bearings, two-row	0.241	0.290	1.250	1.480
Adjustable ball bearings	0.217	0.238	1.070	1.330
Cylindrical roller	0.205	0.257	0.970	1.240
Spherical roller	0.259	0.289	1.150	1.360
Spherical roller, adjustable	0.233	0.278	1.150	1.400
Tapered roller	0.220	0.280	1.300	1.600
Needle roller bearings, cage-less	0.130	0.210	1.570	1.570
Needle roller bearings, with cage	0.130	0.210	0.780	1.000
Axial bearings				
Ball	0.318	0.386	1.190	1.420
Spherical roller, adjustable	0.237	0.253	1.070	1.120
Oblique axial ball bearings	0.340	0.380	1.230	1.410
Cylindrical roller	0.270	0.350	0.850	1.200

D … outer diameter of bearing,

d … inner diameter of bearing.

The bearing distance is optimal when the sum of deformations is minimal – determined graphically:



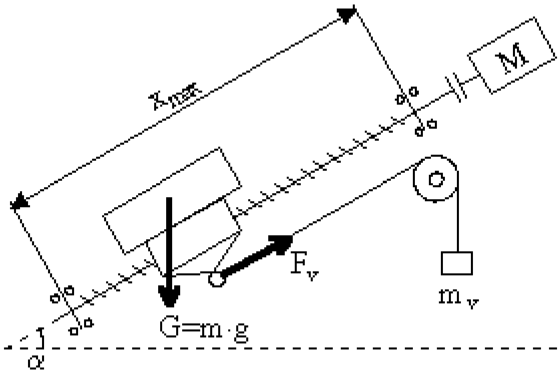
Usually $(L/a)_{opt} \in \langle 2; 6 \rangle$.

6 Calculation of lead screw

Design a lead screw for the given kinematic drawing of a drive and calculate the required torque and its drive.

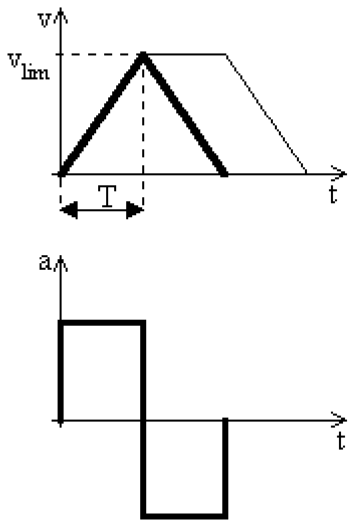
Also, the following is given:

- support weight m [kg],
- counter-weight m_v [kg],
- maximum stroke x_{max} [mm],
- rapid feed v_r [m/min],
- working speed v_p [m/min],
- acceleration a [m/s^2],
- cutting speed F_c [kN],
- coefficient of friction f,
- table tilt angle α [°].



6.1 Rapid feed rate check

e.g. for acceleration ramp:



$$v_{\lim} = a \cdot T$$

$$\underline{x_{\max} = v_{\lim} \cdot T}$$

$$\Rightarrow \quad v_{\lim} = \sqrt{3{,}6 \cdot a \cdot x_{\max}} \quad [m/\min]$$

where: a ... support acceleration [m/s^2],

x_{max} … max. stroke [mm].

The following must apply for rapid feed rate:

$$\underline{v_r \leq v_{\lim}}$$

6.2 Drawing of feed screw

The screw design is to be based on loading force applied on the screw:

$$F_{LOAD} = F_C + F_F + |F_G| + F_d \quad \text{, kde}: \quad F_F = G \cdot \cos\alpha \cdot f; \quad G = m \cdot g;$$
$$F_G = G \cdot \sin\alpha - F_v; \quad F_v = m_v \cdot g;$$
$$F_d = m \cdot a.$$

(absolute value of F_G - always considered for worst case scenario)

This F_{LOAD} force must not exceed the allowed for when loading the screw on buckling.

$$\Rightarrow \quad F_{LOAD} \le \frac{f_u \cdot \pi^2 \cdot E \cdot I}{x_{max}^2 \cdot k} \quad [N]$$

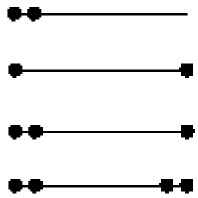
where: f_u ... screw seating coefficient:

$f_u = 0.25$ fixed end - free end

$f_u = 1.0$ both ends buckled (supported)

$f_u = 2.0$ fixed end - supported end

$f_u = 4.0$ fixed end - fixed end



E ... elastic modulus [MPa],

I ... quadratic section modulus $I = \frac{\pi}{64} \cdot d^4$ [mm^4],

d ... Small diameter of screw thread [mm],

x_{max} ... maximum stroke [mm],

k ... safety coefficient (1.3 to 5).

The following table lists allowed tensile / pressure load for short screws:

Screw diameter [mm]	16	18	20	25	28	32	36	40	45	50	55	63	70	80	100
Allowed load [kN]	21	28	32	54	66	82	110	137	184	221	285	338	435	616	1000

Using the previous formula, a ∅d of the screw can be formulated:

$$d = \sqrt[4]{\frac{64 \cdot x_{max}^2 \cdot k \cdot F_{LOAD}}{\pi^3 \cdot f_u \cdot E}} \quad [mm]$$

According to the calculated ∅d, select the corresponding ball screw from the catalogue (watch out for units in some catalogues – e.g. 1kgf = 10N).

Then calculate the maximum allowable revs (during rapid feed) for the designed screw:

$$n_{crit} = \frac{60 \cdot 10^6 \cdot \lambda^2}{2 \cdot \pi \cdot x_{\max}^2} \cdot \sqrt{\frac{E \cdot I}{\rho \cdot A}} \quad [\min^{-1}]$$

where: λ ... screw seating coefficient:

- λ = 1.875 fixed end - free end
- λ = 3.142 both ends buckled (supported)
- λ = 3.927 fixed end - buckled (supported) end
- λ = 4.730 fixed end - fixed end

x_{max} … maximum stroke [mm],

E ... elastic modulus [MPa],

I … quadratic section modulus [mm^4],

ρ ... density, for steel $\rho = 7850 \quad [kg \cdot m^{-3}]$,

A ... screw core section $A = \dfrac{\pi \cdot d^2}{4} \quad [mm^2]$.

Additionally, service life of the screw is calculated:

$$L = \left(\frac{Ca}{f_w \cdot F_{LOAD}} \right)^3 \cdot 10^6 \quad [\text{revs}]$$

where: *Ca* ... basic dynamic capacity (listed in the catalogue) [N],

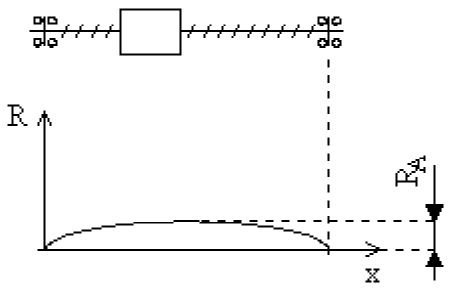
f_w ... loading factor:

- f_w = 1.0 ÷ 1.2 smooth, shock-less motion,
- f_w = 1.2 ÷ 1.5 common motion,
- f_w = 1.5 ÷ 2.0 shock-less motion.

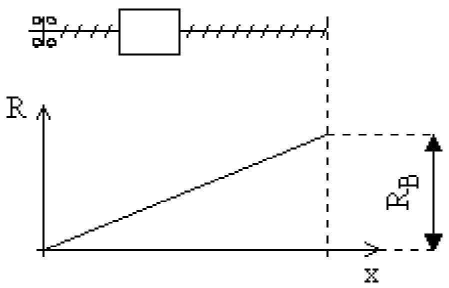
Such designed screw must be inspected as a part of machine servo-drive (i.e. natural frequency must be higher than 50 Hz, the total clearance must not exceed 10 μm).

6.2.1 Calculation of k_1 screw stiffness according to the seating type

A. bilateral (both ends fixed)



B. unilateral screw seating



$$R_A = \frac{L}{4 \cdot E \cdot A} \cdot 10^3 \quad \left[\mu m \cdot N^{-1}\right]$$

$$R_B = 4 \cdot R_A = \frac{L}{E \cdot A} \cdot 10^3 \quad \left[\mu m \cdot N^{-1}\right]$$

where: *R* ... screw elasticity [μm · N^{-1}],

L ... screw length (L = x_{max} + nut length) [mm],

E ... elastic modulus [MPa],

A ... screw section [mm^2].

Screw stiffness: $\Rightarrow \quad k_1 = \frac{1}{R_{A,B}} \quad \left[N \cdot \mu m^{-1}\right]$

6.2.2 Calculation of screw - nut stiffness

$$k_2 = i \cdot d \cdot \chi \quad \left[N \cdot \mu m^{-1}\right]$$

where: *i* ... amount of threads on the nut (from catalogue),

d ... screw diameter [mm]

ϰ ... coefficient, ϰ = 5 [N · mm^{-1} · μm^{-1}].

6.2.3 Calculation of axial bearing stiffness

- see chapter 5 and bearing catalogue $\quad k_3 = \frac{F_{LOAD}}{\delta_a} \quad \left[N \cdot \mu m^{-1}\right]$

Total stiffness of the feed: $\quad k_{total} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}} \quad \left[N \cdot \mu m^{-1}\right]$

6.2.4 Natural feed frequency

$$f_0 = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{k_{total}}{m}} \quad [Hz]$$

where: k_{total} … total feed stiffness [N · m^{-1}],

m … weight of support [kg].

$\underline{f_0 > 50\ Hz}$ (min. 30 Hz – large machines) – large support mechanics condition (1)

f_0 > 1000 Hz – small mechanics (for – scale, sensor assembly)

6.2.5 Loss of motion

Feed deformation when accelerating, manifested for measuring as a clearance.

$$h_k = \frac{2 \cdot F_F}{k_{total}} \quad [\mu m]; \quad [N;\ N \cdot \mu m^{-1}]$$

$\underline{h_k + h_m < 10\ \mu m}$ (max. 20 μm) condition (2)

where: h_k ... stiffness based clearance [μm]

h_m ... mechanical clearance [μm]

If condition (1) is met and condition (2) is met, then the screw **MEETS THE CONDITIONS**.

6.2.6 Drawing of screw pitch

- pitch is determined by maximum allowed n_{safe} screw revs (either found in catalogue of calculated, when $n_{safe} = 0{,}8 \cdot n_{crit}$ $[\min^{-1}]$)

$$n_{\max} = \frac{v_r}{s_s} \quad [\min^{-1}]$$

where: v_r … rapid feed rate [m·min^{-1}],

s_s ... screw pitch [m].

The following condition must be met: $\underline{n_{\max} \le n_{safe}}$

Otherwise, the screw pitch must be increased.

6.3 Drawing the drive

Screw ratio: $K_S = \frac{dx}{d\varphi} = \frac{s_s}{2 \cdot \pi} \quad \left[m \cdot rad^{-1}\right]$

Reducing screw parameters:

$$\frac{1}{2} \cdot I_{red} \cdot \omega_M^2 = \frac{1}{2} \cdot I_S \cdot \omega_M^2 + \frac{1}{2} \cdot m \cdot v_r^2 \quad \Rightarrow \quad \underline{I_{red} = I_S + m \cdot K_S^2} \quad \left[kg \cdot m^2\right]$$

where: I_{red} … reduced moment of inertia,

I_S … moment of inertia of the screw, $I_S = \frac{\pi \cdot d^4 \cdot L}{32} \cdot \rho \quad \left[kg \cdot m^2\right]$,

d … screw diameter [m],

L … screw length [m],

ρ … density [kg·m^{-3}],

m … support mass [kg].

Screw friction torque: $M_F = F_F \cdot K_S \quad [N \cdot m]$,

Static torque: $M_{STAT} = F_G \cdot K_S \pm M_F \quad [N \cdot m]$,

Working torque: $M_W = F_r \cdot K_S + M_{STAT} \quad [N \cdot m]$,

Dynamic torque: $M_{DYN} = M_W + I_{red} \cdot \varepsilon \quad [N \cdot m], \quad \varepsilon = \frac{a}{K_S} \quad \left[s^{-2}\right]$

Required drive power: $P_{RD} = M_{DYN} \cdot \omega_{\max} = M_{DYN} \cdot 2 \cdot \pi \cdot n_{\max} \quad [W]$

where: n_{max} …screw revs during rapid feed [s^{-1}].

The following conditions must be set for drive selection:

$$I_D \geq I_{red},$$
$$0{,}2 \cdot M_N \geq M_F,$$
$$M_N \geq M_W,$$
$$M_N \geq M_{DYN},$$
$$P_D \geq P_{RD},$$

where: I_D ... drive inertia torque [kg·m^2],

M_N ... persistent (nominal) drive torque [N·m],

P_D … power of the drive [W].

Note:

The calculated feed screw is for orientation, more accurate calculations are given by each manufacturer in his ball bearing catalogue.

7 Whitworth mechanism

To obtain parameters for whitworth mechanism, calculate the following:

- required revs of the drive n,
- drive torque M_k ,
- required drive power P,
- ram reverse speed v_R ,
- primary time t_p ,
- secondary time t_s .

When given:

- mechanism dimensions e, r, R,
- table speed v_s ,
- table load F_s .

alternative: Determine the ram working speed and maximum ram cutting power from the given crank revs and motor power.

7.1 Calculation procedure

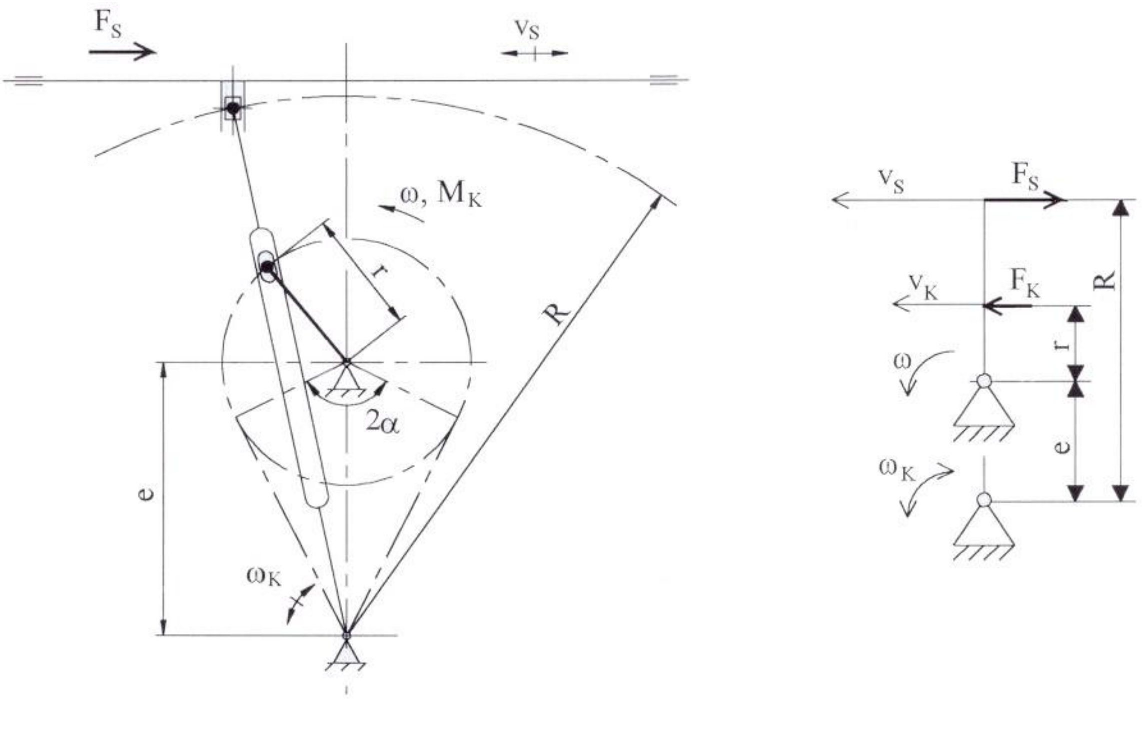


Fig. 5: Whitworth mechanism drawing

7.1.1 Calculation of required drive revs

According to Fig. 5:

$$v_k = \omega \cdot r = \omega_k \cdot (e + r) \quad kde: \quad \omega = 2 \cdot \pi \cdot n \quad a \quad \omega_k = \frac{v_s}{R}$$

$$\Rightarrow \quad n = \frac{v_s \cdot (e + r)}{2 \cdot \pi \cdot r \cdot R} \quad [\min^{-1}]$$

7.1.2 Calculation of drive torque

$$F_S \cdot R = F_K \cdot (e + r) \Rightarrow F_K$$

$$M_{K\max} = F_K \cdot r = \frac{F_S \cdot r \cdot R}{e + r} \quad [\mathrm{N} \cdot \mathrm{m}]$$

7.1.3 Calculation of drive power

$$P = M_{K\max} \cdot \omega = M_{K\max} \cdot 2 \cdot \pi \cdot n \quad [\mathrm{W}]$$

7.2 Example

Given: $v_s = 50\ \mathrm{m} \cdot \min^{-1}$; $F_S = 1000\ \mathrm{N}$; $R = 700\ \mathrm{mm}$; $r = 150\ \mathrm{mm}$; $\mathrm{e} = 250\ \mathrm{mm}$;

Calculate:

$n = 30{,}3\ \min^{-1}$; $M_K = 262{,}5\ \mathrm{N} \cdot \mathrm{m}$; $P = 832{,}9\ \mathrm{W}$;

max. ram reverse speed $v_R = 200\ \mathrm{m} \cdot \min^{-1}$; primary time $t_p = 1{,}4\ \mathrm{s}$; secondary time $t_s = 0{,}58\ \mathrm{s}$.

8 Geneva drive (Maltese cross) mechanism – outer

For the given mechanism, calculate the required motor revs (n), drive torque M_K and required drive power P. When given:

A. secondary time t_s ,

dimensions of mechanism e, r,

table parameters øD, h, ρ,

calculate amount of positions j.

B. secondary time t_s ,

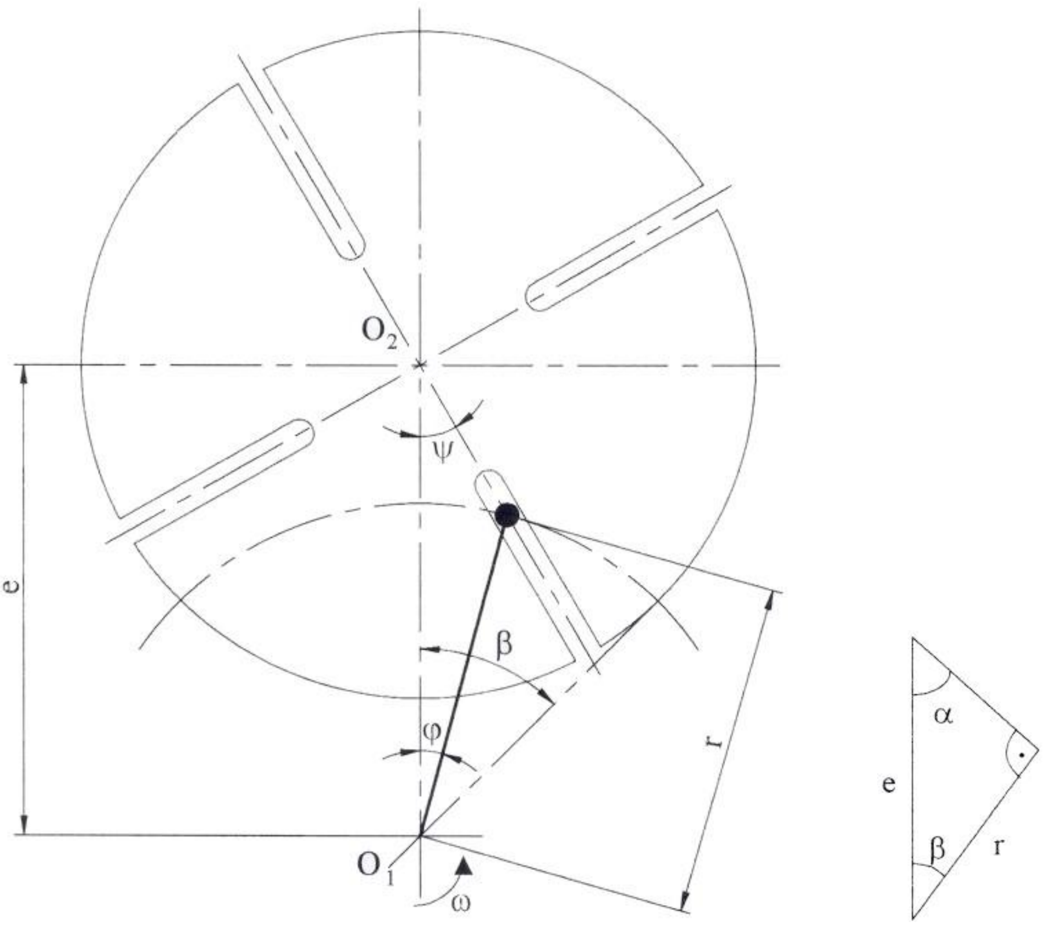
distance of axes e,

amount of positions j,

table parameters øD, h, ρ,

calculate crank size r.

8.1 Calculation procedure



8.1.1 Calculate mechanism parameters

Number of positions j:

$$\sin\alpha = \frac{r}{e} \quad \Rightarrow \quad \alpha$$

$$2\cdot\alpha = \frac{2\pi}{j} \quad \Rightarrow \quad j$$

Crank size r:

$$2\cdot\alpha = \frac{2\pi}{j} \quad \Rightarrow \quad \alpha$$

$$\sin\alpha = \frac{r}{e} \quad \Rightarrow \quad r$$

8.1.2 Calculation of required drive revs

The table rotates by one positions in secondary time t_S :

$$t_s = \frac{2\cdot\beta}{\omega} \quad \omega = 2\cdot\pi\cdot n; \quad \beta = \arccos\frac{r}{e}; \quad \frac{r}{e} = \lambda \quad \Rightarrow \quad n = \frac{\arccos\lambda}{\pi\cdot t_s}$$

8.1.3 Table inertia torque

$$I_T = \frac{1}{2}\cdot m_T\cdot\left(\frac{D}{2}\right)^2 \quad \left[\mathrm{kg}\cdot\mathrm{m}^2\right]; \quad \text{table mass } m_T = \frac{\pi\cdot D^2}{4}\cdot h\cdot\rho \quad [\mathrm{kg}]$$

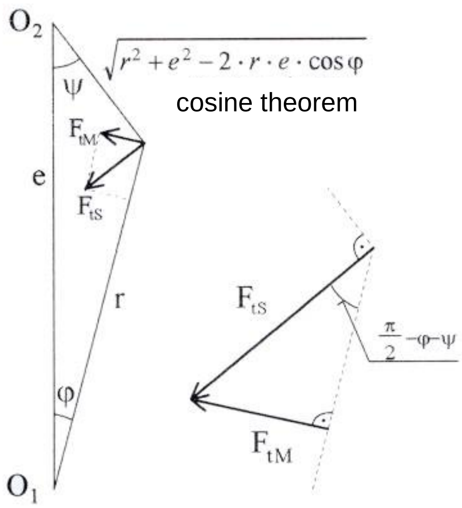
8.1.4 Calculation of drive torque

To rotate a table, it is necessary torque:

$$M_{KT} = I_T\cdot\ddot{\psi} = F_{tS}\cdot\sqrt{r^2+e^2-2\cdot r\cdot e\cdot\cos\varphi}$$

$$\Rightarrow \quad F_{tS} = \frac{I_T\cdot\ddot{\psi}}{\sqrt{r^2+e^2-2\cdot r\cdot e\cdot\cos\varphi}}$$

$$\varphi \in \langle +\beta; -\beta\rangle; \quad \psi = f(\varphi)$$



Using the geometry, determine:

$$\psi = arctg\frac{\lambda \cdot \sin\varphi}{1-\lambda \cdot \cos\varphi} \quad \Rightarrow \quad \dot{\psi} = \frac{\dot{\varphi} \cdot \lambda \cdot \cos\varphi - \dot{\varphi} \cdot \lambda^2}{1-2 \cdot \lambda \cdot \cos\varphi + \lambda^2} \quad \Rightarrow \quad \ddot{\psi} = \frac{\left(\lambda^2 - 1\right) \cdot \dot{\varphi}^2 \cdot \lambda \cdot \sin\varphi}{\left(1-2 \cdot \lambda \cdot \cos\varphi + \lambda^2\right)^2},$$

where $\omega = \dot{\varphi} = \text{const.}$

Drive torque is:

$$M_K = F_{tM} \cdot r, \quad \text{where} \quad F_{tM} = F_{tS} \cdot \sin\left(\frac{\pi}{2} - \varphi - \psi\right) = F_{tS} \cdot \cos(\varphi + \psi)$$

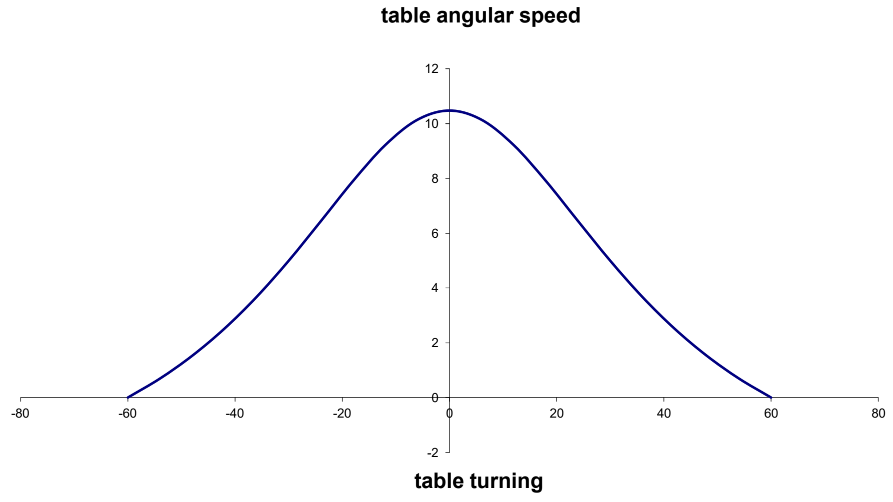
$$M_K = \frac{I_T \cdot \ddot{\psi} \cdot r \cdot \cos(\varphi + \psi)}{\sqrt{r^2 + e^2 - 2 \cdot r \cdot e \cdot \cos\varphi}}$$

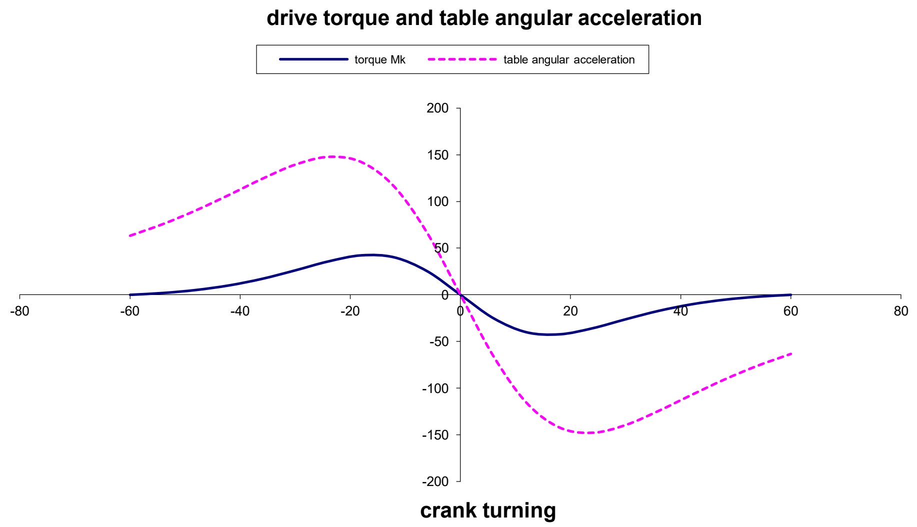
8.1.5 Calculation of drive power

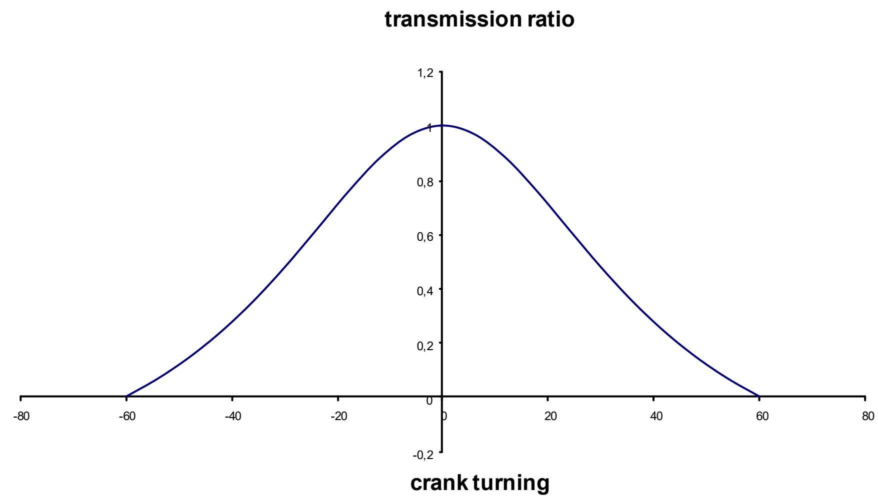
$$P = M_{K\max} \cdot \omega = M_{K\max} \cdot 2 \cdot \pi \cdot n$$

Since the mechanism gearing ratio is not constant, the maximum drive torque M_{Kmax} does not occur during maximum angular acceleration of the table $\ddot{\psi}$. To ensure maximum torque, it is wise to use a mathematical program (e.g. MATHCAD, EXCEL, etc.).

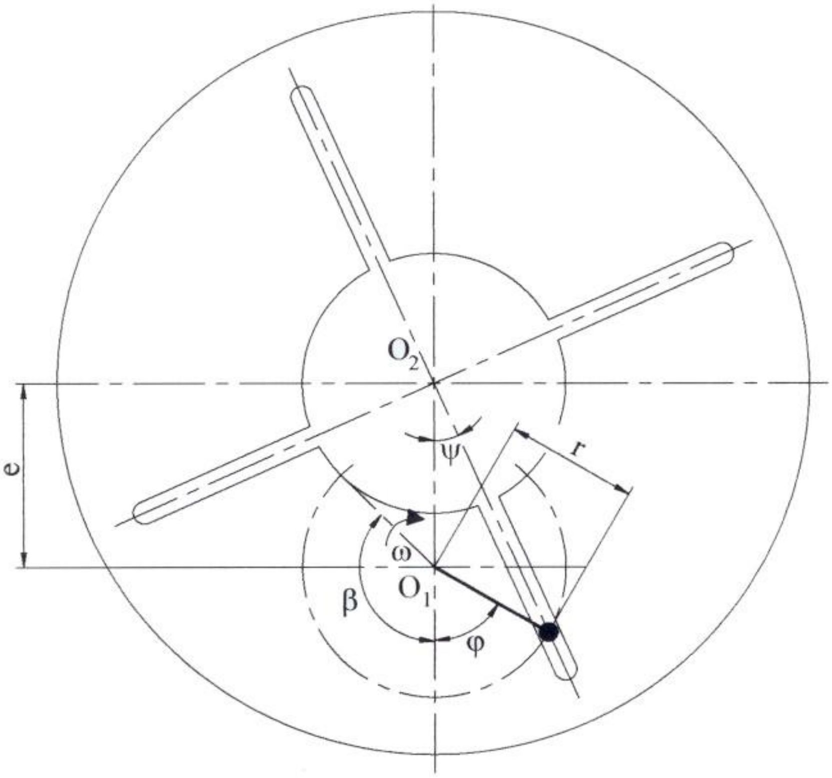
Example for six-position outer mechanism:

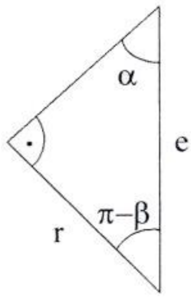






9 Maltese cross mechanism - inner





Calculation is similar to outer Maltese cross mechanism. Calculation of mechanism parameters and calculation of table inertia moment is identical.

9.1 Calculation of required drive revs

The table rotates by one positions in secondary time t_S :

$$t_s = \frac{2 \cdot \beta}{\omega} \quad \omega = 2 \cdot \pi \cdot n; \quad (\pi - \beta) = \arccos\frac{r}{e}; \quad \frac{r}{e} = \lambda \quad \Rightarrow \quad \beta = \arccos(-\lambda)$$

$$n = \frac{\arccos(-\lambda)}{\pi \cdot t_s}$$

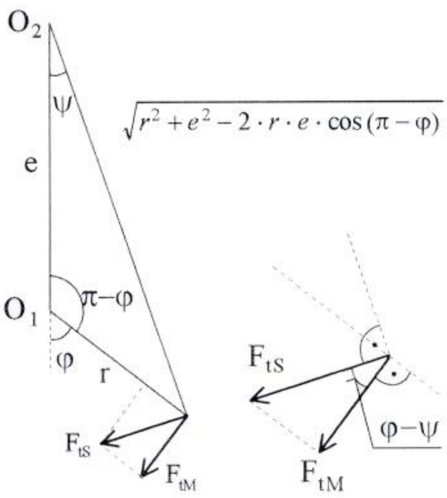
9.2 Calculation of drive torque

For table rotation, a torque is required:

$$M_{KT} = I_T \cdot \ddot{\psi} = F_{tS} \cdot \sqrt{r^2 + e^2 - 2 \cdot r \cdot e \cdot \cos(\pi - \varphi)} \quad \Rightarrow \quad F_{tS} = \frac{I_T \cdot \ddot{\psi}}{\sqrt{r^2 + e^2 + 2 \cdot r \cdot e \cdot \cos\varphi}}$$

$$\varphi \in \langle +\beta; -\beta \rangle; \quad \psi = f(\varphi)$$

Using geometry:



$$\psi = arctg\frac{\lambda\cdot\sin\varphi}{1+\lambda\cdot\cos\varphi} \quad\Rightarrow\quad \dot\psi = \frac{\dot\varphi\cdot\lambda\cdot\cos\varphi+\dot\varphi\cdot\lambda^2}{1+2\cdot\lambda\cdot\cos\varphi+\lambda^2}$$

$$\Rightarrow\quad \ddot\psi = \frac{\left(\lambda^2-1\right)\cdot\dot\varphi^2\cdot\lambda\cdot\sin\varphi}{\left(1+2\cdot\lambda\cdot\cos\varphi+\lambda^2\right)^2},$$

where $\omega = \dot\varphi = \text{const.}$

Drive torque is:

$$M_K = F_{tM}\cdot r,\quad \text{kde}\quad F_{tM} = F_{tS}\cdot\cos(\varphi-\psi)$$

$$M_K = \frac{I_T\cdot\ddot\psi\cdot r\cdot\cos(\varphi-\psi)}{\sqrt{r^2+e^2+2\cdot r\cdot e\cdot\cos\varphi}}$$

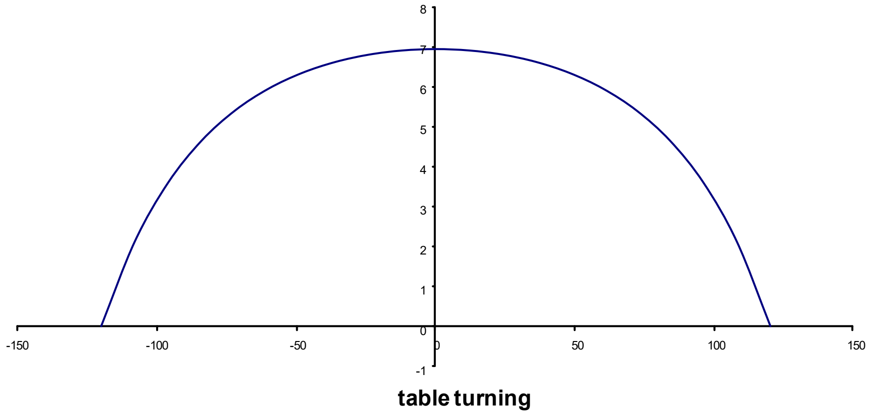
9.3 Calculation of drive power

$$P = M_{K\max}\cdot\omega = M_{K\max}\cdot 2\cdot\pi\cdot n$$

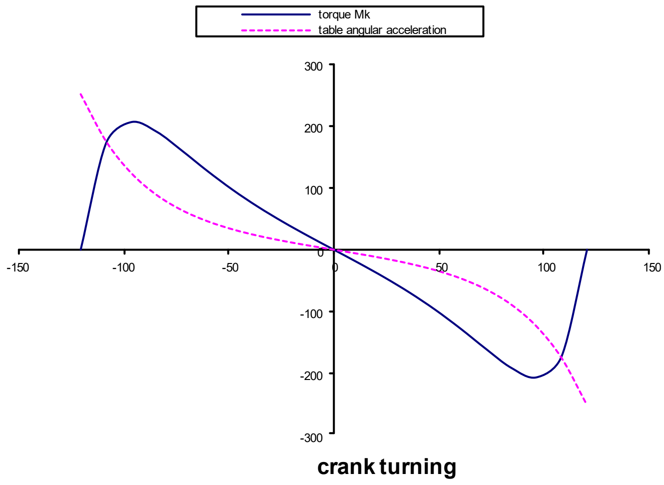
As in the previous case, due to inconstant gearing ratio of the mechanism, the maximum drive torque in the moment of maximum table acceleration is not in the maximum, therefore it is, again, wise to use a mathematical program (MATHCAD, EXCEL, etc.).

Example for six-position mechanism:

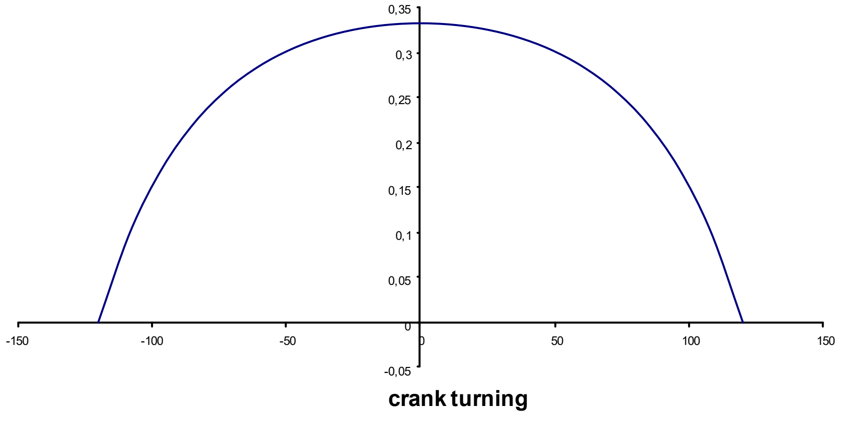
table angular speed



drive torque and table angular acceleration



transmission ratio



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EXAMPLES OF MACHINE PARTS CALCULATIONS

This educational text was created based on long-standing experience with research and education of manufacturing machines, and uses the latest knowledge in this field.

The purpose of this text is for Production Machines I for students of the first year of N2301 Mechanical Engineering follow-up study program, branch 2302T010 Machines and Equipment Design, field Production Machines.

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