

# Optimization of machine spindle mounting

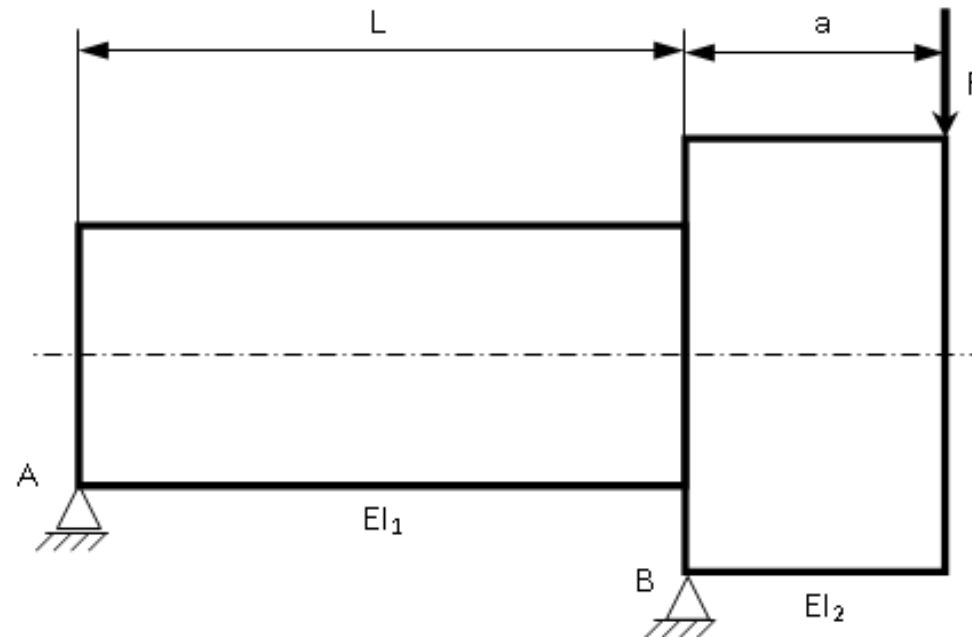
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## Task

For a given load and the characteristic dimensions of the machine tool spindle (see sketch), calculate the optimal distance of bearings. Additionally redraw the drawing of spindle in scale 1 : 1.

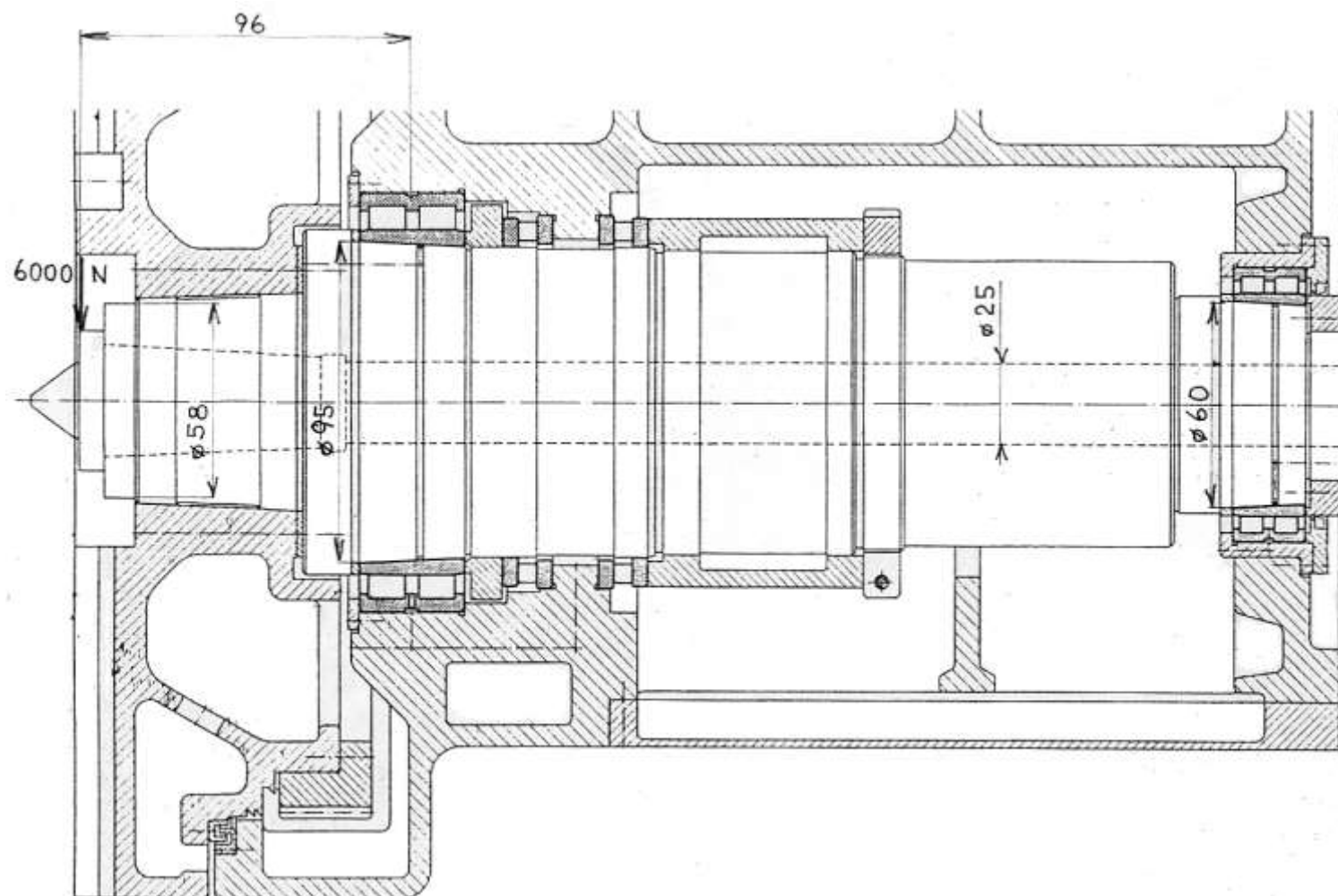
Sketch of the computational model of spindle :



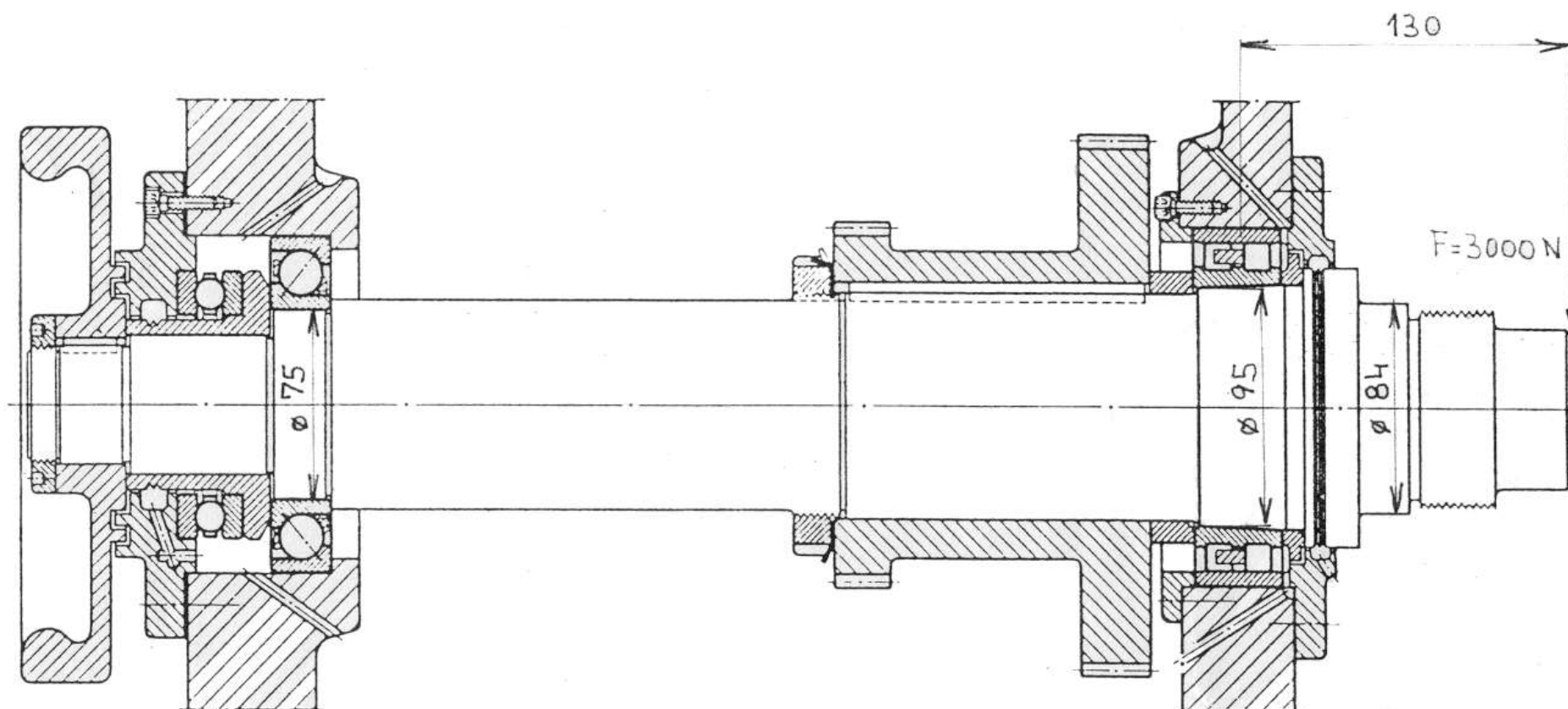
**Given:**  $F$ ,  $a$ , some characteristic dimensions of spindle

**Identify:**  $L_{opt}$

## Specific examples of task:



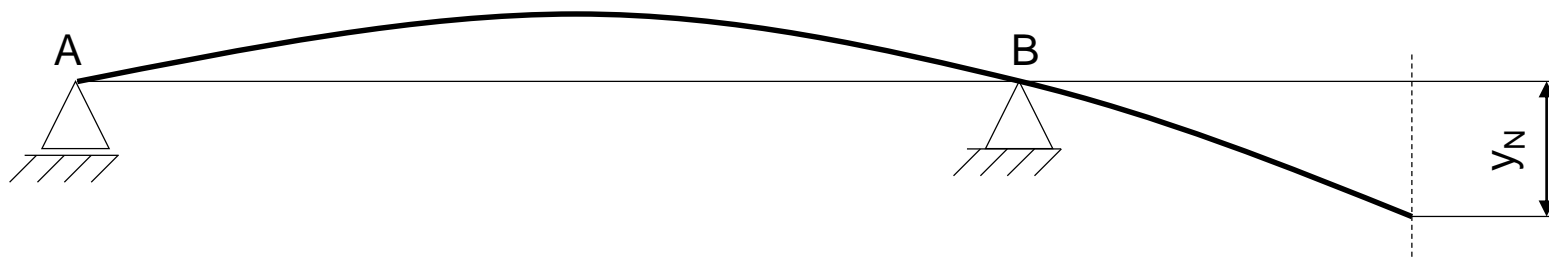
## Specific examples of task:



## The calculation procedure

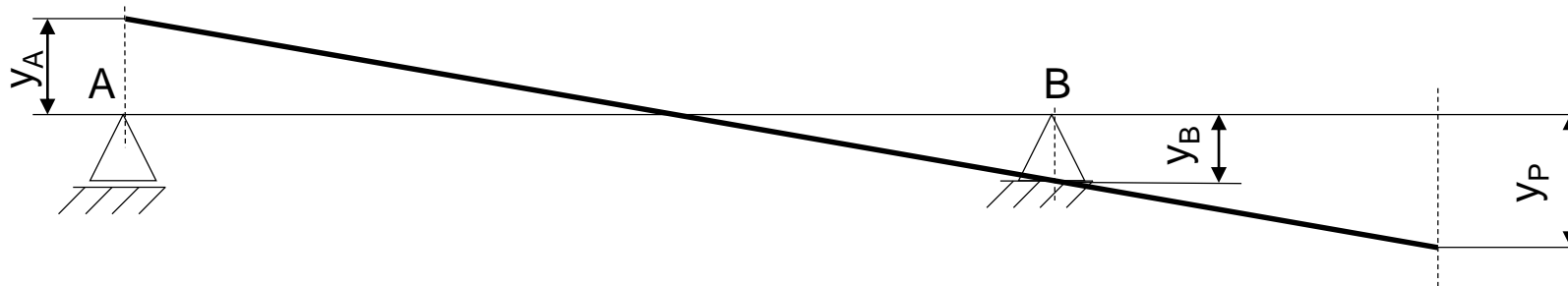
The total deflection of spindle is the sum of deformation of spindle and deformation of bearings.

1. Deflection of spindle loaded by force  $F$  provided that the bearings are stiff:



$$y_N = \frac{Fa^2L}{3EI_1} + \frac{Fa^3}{3EI_2}$$

## 2. Deflection of spindle, provided that the bearings are compliant and stiff spindle:



$$\frac{y_A + y_P}{a + L} = \frac{y_A + y_B}{L} \quad \Rightarrow \quad y_P = \frac{(y_A + y_B) \cdot (a + L)}{L} - y_A$$

Elastic deformation of bearings  $y_A$  ,  $y_B$  is determined by the following table:

bearing type	loading specifications	
	$\delta_a = 0$	$\delta_r = 0$
self-aligning ball bearings	$\delta_r = \frac{70 \cdot 10^{-5}}{\cos \alpha} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$	-----
radial ball bearings	$\delta_r = 44 \cdot 10^{-5} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$	-----
ball bearings, angular contact	$\delta_r = \frac{44 \cdot 10^{-5}}{\cos \alpha} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$	$\delta_a = \frac{44 \cdot 10^{-5}}{\sin \alpha} \cdot \sqrt[3]{\frac{Q^2}{D_W}}$

Elastic deformation of bearings  $y_A$  ,  $y_B$  is determined by the following table:

bearing type	loading specifications	
	$\delta_a = 0$	$\delta_r = 0$
Bearings with straight line contact on both rings	$\delta_r = \frac{8 \cdot 10^{-5}}{\cos \alpha} \cdot \frac{Q^{0,9}}{L_a^{0,8}}$	$\delta_a = \frac{8 \cdot 10^{-5}}{\sin \alpha} \cdot \frac{Q^{0,9}}{L_a^{0,8}}$
Bearings with straight line contact on one ring and point contact on the second ring	$\delta_r = \frac{22 \cdot 10^{-5}}{\cos \alpha} \cdot \frac{Q^{3/4}}{L_a^{1/2}}$	$\delta_a = \frac{22 \cdot 10^{-5}}{\sin \alpha} \cdot \frac{Q^{3/4}}{L_a^{1/2}}$
Thrust ball bearings	-----	$\delta_a = \frac{52 \cdot 10^{-5}}{\sin \alpha} \cdot \sqrt[3]{\frac{Q^2}{D_w}}$
The force at rolling element	$Q = \frac{5 \cdot F_r}{i \cdot z \cdot \cos \alpha}$	$Q = \frac{F_a}{z \cdot \sin \alpha}$



where:

$i$	Number of rows of rolling elements	
$z$	Number of rolling elements in one row	
$\delta_r$	Deformation in the radial direction	[mm]
$\delta_a$	Deformation in the axial direction	[mm]
$D_W$	Diameter of the rolling elements	[mm]
$L_a$	Effective length of the rol. elements	[mm]
$\alpha$	Contact angle	
$F_r$	Load in the radial direction	[N]
$F_a$	Load in the axial direction	[N]
$Q$	Force at rolling element	[N]

If there are no dimensions of rolling elements and their number given in the catalogue, can be approximately calculated:

$$D_W = q_1 \cdot (D - d)$$

$$z = q_2 \cdot \frac{D + d}{D_W}$$

$$L_a = 1,4 \cdot D_W$$

where:

- $D$  bearing outside diameter
- $d$  inner diameter of the bearing

## Table for calculating the approximate dimensions of rolling element bearings:

Bearing type	q <sub>1</sub>		q <sub>2</sub>	
	from	to	from	to
<b>Radial bearings</b>				
Ball, single row	0,216	0,330	0,890	0,990
Ball, double row	0,200	0,280	1,190	1,390
Ball bearings, angular contact, single row	0,250	0,320	1,240	1,400
Ball bearings, angular contact, double row	0,241	0,290	1,250	1,480
Self-aligning ball bearings	0,217	0,238	1,070	1,330
Cylindrical roller bearings	0,205	0,257	0,970	1,240
Spherical roller bearings	0,259	0,289	1,150	1,360
Tilting spherical roller bearings	0,233	0,278	1,150	1,400
Tapered roller bearings	0,220	0,280	1,300	1,600
Needle roller bearings without cage	0,130	0,210	1,570	1,570
Needle roller bearings with cage	0,130	0,210	0,780	1,000

Table for calculating the approximate dimensions of rolling element bearings:

Bearing type	$q_1$		$q_2$	
	from	to	from	to
<b>Axial bearings</b>				
Ball	0,318	0,386	1,190	1,420
Tilting spherical roller bearings	0,237	0,253	1,070	1,120
Ball axial bearings, angular contact	0,340	0,380	1,230	1,410
Cylindrical roller bearings	0,270	0,350	0,850	1,200

where:

$q_1$  and  $q_2$  coefficients for calculating the dimensions of rolling elements according to type of bearings

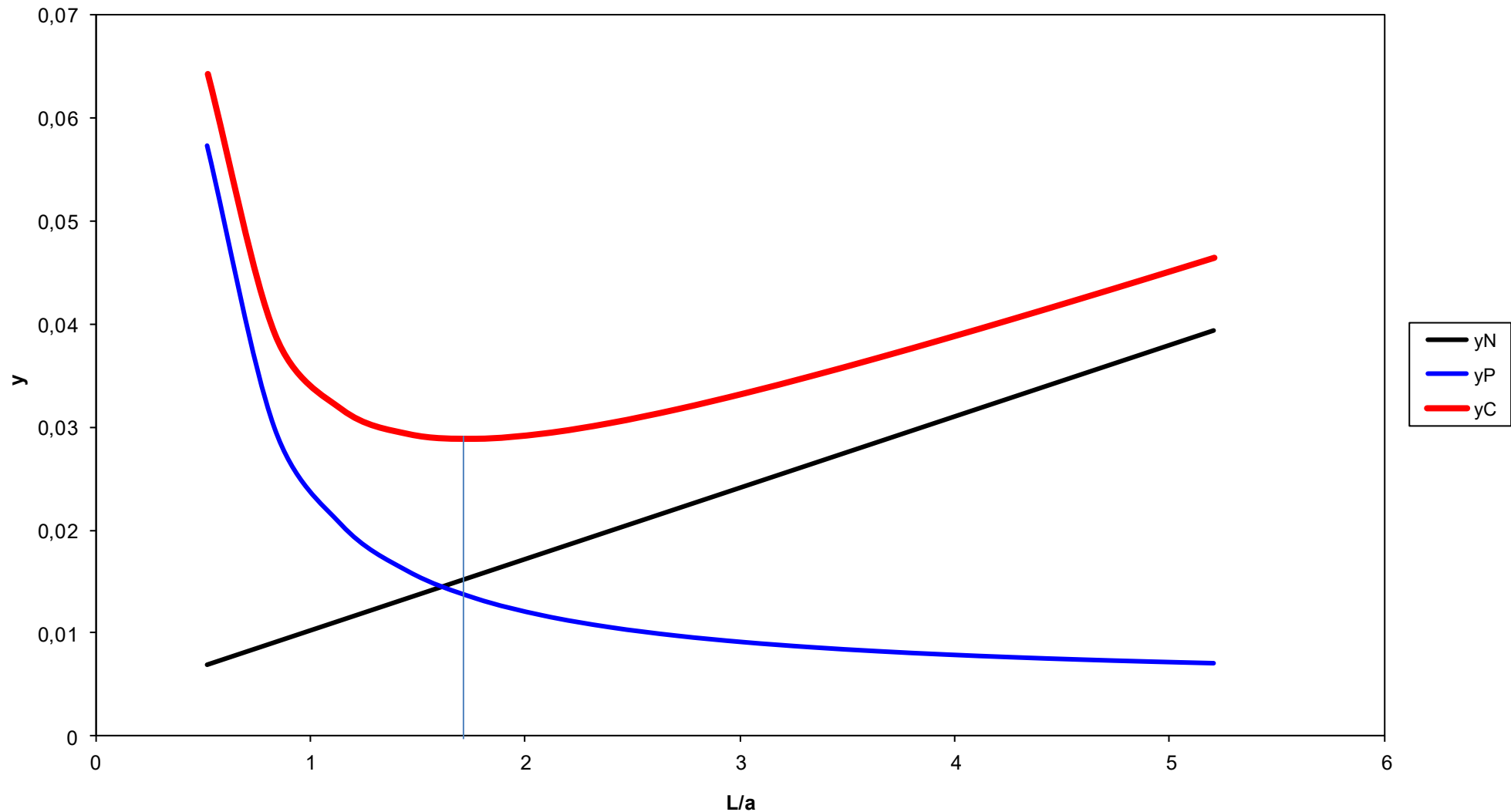
The optimal distance spindle bearings is in the case where the sum of deformations is minimal – it is determined graphically as sum  $y_P$  and  $y_N$  depending on searched bearings distance  $L$  or ratio  $L/a$ .

$$y_C = y_N + y_P$$

$$y_N = \frac{Fa^2L}{3EI_1} + \frac{Fa^3}{3EI_2}$$

$$y_P = \frac{(y_A + y_B) \cdot (a + L)}{L} - y_A$$

The optimal distance of spindle bearings is in the case where the sum of the deformations is minimal - that is determined graphically:  
 Spindle deflection



The optimal distance of spindle bearings is in the case where the sum of the deformations is minimal – most often

$$(L/a)_{\text{opt}} \in \langle 2; 6 \rangle$$